M. N. RASTOGI

THE THEORIES OF IMPLICATION IN INDIAN AND WESTERN PHILOSOPHY

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(A CRITICAL STUDY)

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By

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In loving and sacred memory

of

my mother

Shrimati Shyam Dei Rastogi

and

my father

Shri Kunj Behari Lal Rastogi

Foreword

This work presents an original conception and analysis of implication in the sense of entailment. It proceeds in both historical and critical fashion. After distinguishing the relation of implication from the process of inference, the author examines the relevant ideas of Aristotle and those involved in the Nyāya Syllogism. The logical doctrines of Absolute Idealism are next considered with special reference to the writings of Bosanquet. They are criticized primarily for their connection with the unsatisfactory coherence theory of truth. The notion of material implication is examined carefully and found not to be a satisfactory model of what we mean by implication as the relation between premises and conclusion in a valid argument. C.I. Lewis's strict implication is considered with some sympathy, but is also ultimately rejected for good cause. The analyses of a number of other contemporary writers on logic are examined, including E.J. Nelson, J. Bennet, Strawson, Geach, Hampshire, and Korner. From the shortcomings of earlier treatments we are led to the new definition of implication in terms of systems of thought and their constituents. The new conception of implication does indeed seem to be free from the difficulties connected with earlier treatments of it. The work represents a considerable amount of research and hard, careful thinking.

University of Hawaii
July 31, 1980

Irving M. Copi

Preface

As a student and teacher, the Problem of Implication appeared to me to be the central problem of logic. I, therefore, took this problem for my doctoral thesis of the Lucknow University. The present work is the revised form of that thesis. In this work my aim has been to state and examine different Theories of Implication —Indian and Western—in order to distinguish them from *ordinary implication*, and to attempt to develop my theory of Systematic Intensional Implication which is what is *ordinarily* meant by 'implication' when we say, with regard to any ordinary deductive reasoning, that the premises imply the conclusion. The theory is mainly developed in Chapters VII to X. It is the relation of ordinary implication which is expressed by Moore by the term 'entailment' when he says that 'it is the converse of that relation which we assert to hold between a particular proposition *q* and a particular proposition *p*, when we assert that *q follows from* or is *deducible from p*.'

Readers may differ with my views on 'Implication'. But it is in such differences that the charm of philosophy lies and it is such differences that lead to the growth of the subject.

To express my views I have introduced some new symbols. I have used 'I' for 'ordinary or systematic intensional implication', '$\leftarrow\!\!\twoheadrightarrow$' for 'has as its constituent', '$i \sim \Diamond$' for 'intensional impossibility', '*fP*' for 'the (corresponding) fact *P*', and 'C' for 'containment'. The symbol '$\leftarrow\!\!\twoheadrightarrow$' is used in the ideal sense, that is, when *p* implies *q*, the meaning or *thought-system* expressed by *p* 'has as its constituent' the meaning or *thought* expressed by *q*. As logicians have often used '*p*' for '*p* is true' and '$\sim q$' for '*q* is false', it has not been possible for me to use such symbols otherwise while expressing their views. But when expressing my view of systematic intensional implication, I have used '*p*' for 'the *proposition p*' and '$\sim q$' for 'the *proposition* which is contradictory of *q*',

irrespective of their truth-values. The exact meaning of these symbols in each case can be understood with reference to the particular context.

Logicians have sometimes spoken of 'the meanings expressed by the sentences' and propositions are said to be the meanings of declarative sentences. But in this work I am speaking of 'the meaning expressed by a proposition' because there is a difference of opinion among the logicians regarding the exact nature of a proposition and what it exactly expresses. What a proposition *exactly expresses* is what I call the meaning of that proposition and this meaning is ideal and not factual. Proposition is the meaning of a sentence but we can speak of the meaning of the meaning of a sentence.

It is a pleasure to express my indebtedness to the eminent scholars and thinkers whose views I have studied and discussed in this work. I am indebted to Dr. Raj Narain, retd. Professor of Philosophy, Lucknow University, for his valuable suggestions and encouragement. Prof. G. Ryle sent from Oxford valuable suggestions and comments on a brief abstract of this work. I am thankful to him.

I am deeply obliged to Pt. Bhawani Shanker with whom I have been having valuable and interesting discussions on many philosophical topics.

My thanks are due to Messrs. Bharatiya Vidya Prakashan, Delhi, for undertaking the publication of this work. In spite of the best care, a few misprints have crept in for which we crave the indulgence of the readers. Corrigenda will, however, help minimising their inconvenience.

Maharaj Narain Rastogi

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Introduction : Inference and Implication

The Sanskrit word for inference is 'Anumāna' which literally means knowledge (Māna) after (Anu) something. It is mediate knowledge derived through the medium of some other knowledge. But the derivation of such mediate knowledge always involves a psychological-cum-logical process. Therefore the word 'Anumāna' is used for both the psychological-cum-logical process and the mediate knowledge derived from it. Stebbing defines inference as 'a mental process in which a thinker passes from the apprehension of something given, the datum, to something, the conclusion, related in a certain way to the datum, and accepted only because the datum has been accepted.'[1] But we must note that the mental process involved in inference is based on certain logical relations between different thoughts.

Inference is to be distinguished from other mental processes such as perception, recollection and imagination. Suppose a man sees a red object and recognizes it as a post-box. Again he sees a moving object and recognizes it as a bull. Such judgments are judgments of perception. They refer to the present objects and are immediate. They are to be distinguished from inferential judgments which are mediate. Perception being immediate can refer only to the present, while inference may refer to the past and future also.

Recollection is the revival of a past experience which has been conserved in the mental structure in the form of a psychical disposition. In it we do not pass from one knowledge to another but only revive past perceptual knowledge. As against it, in

1. *A Modern Introduction to Logic* (1946), p. 212.

inference we pass from one knowledge to another, from the datum to the conclusion.

In imagination the mind recalls the elements of past experience and combines them into a new pattern. A man lying on the bank of the Ganges may indulge in an idle reverie suggested by his recollections of past events. This is random thinking involving free association and is to be distinguished from logical thinking which depends upon logical relations between different thoughts and is of the form: "This, therefore, that."

The word 'inference' is wide enough to include not only deduction but also induction. As stated above, inference is any passage from a datum to a conclusion. This notion of inference includes induction also because in induction, as usually understood, we pass from the knowledge of some instances of a class to the knowledge of all instances of that class. Whether an inference is deductive or inductive depends upon the *nature of the relation* that holds between the given propositions and the conclusion. We must, therefore, clearly understand the nature of this relation.

In deductive inference the premises *imply* the conclusion, while in inductive inference the premises *support* or are the *evidence* for the conclusion. Sometimes the word 'inductive' tends to be reserved for those cases in which the conclusion is a general statement. But such a restriction seems to be artificial. For the assertion of a non-necessary general proposition on the basis of some evidence is the same as the assertion of a particular proposition on the basis of some other evidence. For example, the assertion of the proposition that 'all men are mortal' on the basis of the evidence that 'Rāma, Krishṇa, Mohan...and John are mortal', is the same as the assertion that 'it will rain here next week' on the basis of the evidence that 'monsoon is moving towards Uttar Pradesh', or the assertion that 'Kālidāsa lived in the 4th century A.D.' on the basis of some historical evidence collected. In all such cases, it would be proper to say that the premises *support* or are the *evidence* for the conclusion but do *not imply* it. Where the premises imply the conclusion, the latter *necessarily* follows from the former, and it would be self-contradictory to say $p.{\sim}q$[1] (where 'p' expresses the premises and 'q' the

1. Here I am using 'p' for 'the proposition p' and not for 'p is true'; similarly '${\sim}q$' for 'the proposition which is contradictory of q', for I hold that the relation of implication holds between propositions, irrespective of their truth-values.

conclusion). As against it, in induction the conclusion does not necessarily follow from the premises; it is more or less *probable* but never necessary. Hume was right in holding that we cannot demonstrate that a certain cause must have a certain effect or that it must always have the same effect. There is no necessary connection between bread and nourishment such that the one necessarily follows from the other. There is nothing logically contradictory in saying that bread will not nourish or fire will not warm. In his article 'The Logic of Causal Propositions', *Mind*, 1951, Prof. A.W. Burks uses the phrase 'causal implication'. But it is quite obvious that the sense in which a cause *produces* an effect is quite different from the sense in which the premises imply the conclusion. The very fact that Prof. Burks uses the phrase '*causal implication*' and *not* '*implication*' is sufficient to prove that the two notions are different.

If inductive conclusions are not necessary, what is the justification for believing them? What value can be assigned to them?

When the detective, the historian and the scientist draw conclusions from their premises. their evidence, the premises do not imply the conclusions. Judged from the deductive standards, the conclusions may be said to be invalid. But that does not mean that they are unsound or useless. They are more or less probable and serve our practical purpose. Deductive standards are not the only standards of good reasoning. Inductive reasoning has its own standards of validity and it is inappropriate to apply deductive standards to it.

Philosophers since Hume have attempted to avoid the scepticism implicit in Hume's views regarding induction. None can deny that we have *belief* in inductive conclusions. Even Hume accepts that we have such beliefs. We are led to expect upon the appearance of the cause, the appearance of the effect; the mind is carried by *habit* or *custom* to believe. What Hume denies is the logical (deductive) justification for such beliefs. He distinguishes between propositions with regard to matters of fact and propositions with regard to abstract concepts such as those of mathematics. While the latter are logically necessary, the former are not.

Kant tried to solve the problem of induction by assuming the universal and necessary character of the *a priori* categories of understanding to which all our experience must conform. But instead of solving he evades the problem by denying the empirical

character of induction. Another solution seems to be given by those logicians (e.g., Joseph) who believe in the Uniformity of Nature and the Law of Universal Causation. The Law of Universal Causation as well as the Principle of Limited Independent Variety[1] and the Principle of Atomic Uniformity[2] are different forms of the Principle of the Uniformity of Nature. If we accept this principle, then it would be the supreme general proposition which taken in conjunction with the evidence for any accepted generalization of science or daily life would imply that generalization. But the question is how are we justified in asserting that 'nature is uniform'. Is this supreme general proposition true or false? All inductive generalization may be said to be *necessarily true* only *if* this supreme general position is necessarily true. But how do we establish it? It should be noted that it itself is established by an inductive process which is as follows :

> Inductions A, B, C, D......etc. *support* that the nature is uniform,
> Therefore the nature is uniform.

Hence to justify inductive conclusions as *necessary* on the basis of the Principle of the Uniformity of Nature would be circular reasoning. Perhaps such a mistake would not have been committed, had it been recognized that inductive conclusions are not necessary or certain in the same sense in which deductive conclusions are. The Principle of Uniformity of Nature is not a logical but a practical principle. Inductive conclusions are valid from practical standards and are useful for all practical purposes. They may be said to be less or more probable, or conclusively established, depending upon the evidence in their favour but they are never logically necessary.

1. In his book *Treatise on Probability* (1921, p. 258), Keynes says : "That the amount of variety in the universe is limited in such a way that there is no one object so complex that its qualities fall into an infinite number of independent groups (i.e., groups which might exist independently as well as in conjunction); or rather, that none of the objects about which we generalize are as complex as this; or at least that, though some objects may be infinitely complex, we sometimes have a finite probability that an object about which we seek to generalize is not infinitely complex."
2. The Principle of Atomic Uniformity, formulated by Keynes, is the assumption that natural occurrences can be regarded as compounded of small changes taking place in accordance with mathematical laws.

Having discussed the relation between the premises and the conclusion in inductive reasoning, let us now discuss the relation between the premises and the conclusion in *ordinary deductive reasoning*. The relation that exists between the premises and the conclusion in ordinary deductive reasoning is called *implication*. Moore has used the term '*entailment*' for this relation. It is the *converse of that relation* which holds between a particular proposition *p* and a particular proposition *q*, when we assert that *q follows from* or is *deducible from p*. Russell says : "Now in order that one proposition may be inferred from another, it is necessary that the two should have that relation which makes the one a consequence of the other. When a proposition *q* is a consequence of a proposition *p*, we say that *p implies q*. Thus deduction depends upon the relation of *implication*, and every deductive system must contain among its premises as many of the properties of implication as are necessary to legitimate the ordinary procedure of deduction."[1] Here Russell seems to be in agreement with the *ordinary usage* of language when he uses the expression "*p* implies *q*" for the relation that holds between *p* and *q* when *q* is deducible from *p*.

What is meant by implication can be made clear by some examples : 'This is red' implies 'this is coloured'; 'All men are mortal and Ram is a man' implies 'Ram is mortal'. The proposition 'Ram is a man' does not imply 'Ram is mortal', though both are true. It must be noted that the notion of implication as it is *ordinarily* used is different from the notion of material implication used by Russell. Ordinary implication is *a relation between the meanings* of the propositions and we can know it independently of their truth-values. When *p* ordinarily implies *q*, we can assert the relation between the truth-values of the propositions only hypothetically—if *p* is true, *q* is true. But such is not the case with material implication. It is defined as "it is *not the case* that *p* is true and *q* is false." It asserts an actual connection between the truth-values of *p* and *q* and may be said to be a *truth-functional* notion. Prof. Copi has used the phrase 'extremely weak kind of implication'[2] for it. But whatever phrase we may use, it should not be confused with ordinary implication.

1. *Principia Mathematica*, Vol. I (1925), p. 90.
2. *Symbolic Logic* (1961), p. 18.

Implication is the logical relation between the premises and the conclusion in any vaiid deductive inference. Whenever *q* is inferred from *p*, *p* must imply *q*. This is a necessary logical condition for any deductive inference. 'If......then......' is the sign of implication, and '......therefore......' the sign of deductive inference. In order that we may be able to pass from 'If......then ......' to '......therefore......' certain conditions must be fulfilled. Johnson has mentioned two sets of conditions—constitutive and epistemic—for any deductive inference. Constitutive conditions refer to the propositions and the relations that hold between them; epistemic conditions refer to the relation of the propositions to what the thinker may happen to *know*. Both these conditions, as stated by Johnson,[1] are :

Constitutive conditions : (i) the proposition '*p*' and (ii) the proposition '*p* would imply *q*' must both be true.

Epistemic conditions : (i) the asserting of '*p*' and (ii) the asserting of '*p* would imply *q*' must both be permissible without reference to the asserting of '*q*'.

Sometimes the distinction between inference and implication is expressed by calling implication 'hypothetical inference', for in inference the premise is categorically asserted, while in the relation of implication, it is put forward only hypothetically. Since in implication the premise is not asserted, that is, does not express a fact, Malcolm is wrong in explaining implication in terms of facts.[2] Implication is a logical relation between the meanings of the propositions corresponding to which, as we shall discuss latter, there exists the relation of *containment* between facts, *provided* they exist. The *logical structure* of thought corresponds to that of facts. It is in this sense that thought may be said to be the mirror of factual reality.

The mind is purely receptive in perceiving, by natural insight, the relation of implication. It is a logical relation between propositions and exists independently of any thinker. It is an *objective* relation and is not the least affected by our perceiving or

1. *Logic*, Part II (1922), p. 8.
2. Malcolm's view of implication is that the fact expressed by the consequent is identical with that expressed by the antecedent, or at least is 'not a new fact in addition to the first'. See 'The Nature of Entailment', *Mind*, 1940.

not perceiving it. But its perception is necessary for any inference. It is not enough that we assert this relation, we must also assert the premise in order to infer. Russell rightly puts it as follows : "When we say '*therefore*', we state a relation which can hold between *asserted* propositions, and which thus differs from implication. Whenever 'therefore' occurs, the hypothesis (i.e., the implicans) may be dropped, and the conclusion asserted by itself."[1]

1. *Principles of Mathematics* (1956), p. 35.

2

Linear Implication : Aristotle

In the traditional logic it was taken as obvious that a statement or proposition falls naturally into three parts : subject, predicate and copula. A subject is that about which something is asserted, and the predicate is what is asserted about it. The traditional logicians assumed that every proposition contained two, and only two terms, subject and predicate, connected by the copula which is always the verb 'to be'. The logical form of every proposition is 'A is B'. It was supposed that every proposition was analysable into subject and predicate, and must, therefore, be expressed with the help of the verb 'to be'. A proposition such as 'Boys make noise' should be restated in the verbal form : 'Boys are beings who make noise', in order to bring out the logical form : subject-copula-predicate. The distinction of subject and predicate is always present in thought when we judge though sometimes it may be very vague.

The old logic maintains that all inference, except immediate inference, is syllogistic.[1] A syllogism is actually an argument in which, from the given relation of two terms, in the way of subject and predicate, to some third term, there follows necessarily a relation between those two terms themselves. The following is generally given as an example of the Aristotelian Syllogism :

All men are mortal,
Socrates is a man,
∴ Socrates is mortal.

1. Aristotle recognized only one form of inductive inference, viz., 'induction by complete enumeration'. It can be symbolically expressed as follows :
S_1, S_2, S_3...Sn is P,
Every S is either S_1 or S_2 or S_3...or Sn,
∴ Every S is P.

But we must distinguish between a Peripatetic Syllogism and an Aristotelian one. The given example is a Peripatetic one. It is an inference and the characteristic sign of an inference is the word 'therefore'. No syllogism is formulated by Aristotle as an inference, but they are all *implications* having the conjunction of the premises as the implicans and the conclusion as the implicate. He gave the following definition : 'A syllogism is discourse in which certain things being stated, something other than what is stated follows of necessity from their being so.'[1] In accordance with this definition we may say that a syllogism is a form of implication in which two propositions jointly imply a third. A true example of an Aristotelian syllogism would be, therefore, the following implication :

If all men are mortal
and all Greeks are men,
then all Greeks are mortal.

Our main concern is to discuss the syllogistic principle on the basis of which the antecedent *implies* the consequent. It may be mentioned that Aristotle did not regard all figures of the syllogism as in themselves immediately convincing. He recognized only three figures and thought that the validity of the second and third figures was ultimately assured by reduction to the first. But how are we assured of the validity of the first figure? What is the principle of syllogistic necessity? The following is the exact English version of the most important Aristotelian syllogism, later called 'Barbara' :

If A is predicated of all B
and B is predicated of all C,
then A *must* be predicated of all C.

The word 'must' is the sign of syllogistic necessity and is used by Aristotle in almost all implications which contain variables.

The principle of syllogistic necessity has been supposed to be furnished in the *Dictum de omni et nullo*. This principle may be expressed thus—'what is predicated about any whole is predicated about any part of that whole'. Aristotle seems to have thought that we were assured of the validity of the first figure by the fact that it conformed to the *Dictum de omni et nullo*, this dictum

1. *Analytica Priora*, 24b . 18.

itself being a self-evident proposition. An example will make the meaning of the principle quite clear :

If all B is A,
and all C is B,
then all C is A.

Here it does not matter for what real terms A, B and C stand. Whatever they are, the implication is valid. The Dictum is the nerve of all reasoning.

Let us examine the claims of this principle. The Dictum as it is formulated is too vague. 'What is predicated about any whole is predicated about any part of that whole'. What do we mean by a 'whole' here. If it is a class or collection, if the major premise is to be understood in extension, then a syllogism would be a *petitio principii*. It is the main criticism against the Dictum. By *petitio principii* or begging the question is meant assuming in one of our premises what we have to prove. According to this view, the major premise of a syllogism is ascertained by examining all the cases which means that the case asserted in the conclusion is also examined before making the assertion. Taking the above example, if I assert the major premise, 'All B is A', I must certainly have examined C before making the assertion; and, therefore, the major premise, 'All B is A', rests on the conclusion, all C is A. According to this interpretation, the major premise of a syllogism is an enumerative.

There is another interpretation of the Dictum. The major premise may be taken as asserting the connection between universals or concepts. Then the major premise, 'All B is A', is not a collective statement about every B, C...etc. included; it is a true universal judgment in which B is the middle term and the major premise connects being A with being B. Both the major and middle terms are universals. The minor may indeed be an individual, though it need not be so. In the example :

If all men are mortal,
and Rāma is a man,
then Rāma is mortal,

the major term 'mortal' and the middle term 'men' are universals or characters, and the minor term 'Rāma' is an individual. The perception that the middle term is not a class but a character, has

led to the formulation of another principle which seems to be more satisfactory than the *Dictum de omni et nullo*. The principle is this—*Nota notae est nota rei ipsius*, i.e., what qualifies an attribute qualifies the thing possessing it. On the basis of the immediate apprehension of the connection between universals or characters, we apprehend the connection between an individual and a character. Certain objections may be made to it. It speaks as if the connection between universals or characters is of the same type as the connection between a concrete subject and a universal, and the former can be immediately apprehended in the same way as the latter. But really these are two different types of apprehensions. While the latter is possible, there is no justification for believing the former. In accordance with Aristotle's own views there are no universals apart from the particulars. There are no characters but only 'characterised things'. Then how can we apprehend or have intuition of the connection between universals? Moreover, we *cannot pass from* the apprehension of the connection between universals to the apprehension of connection between a concrete subject and a universal as they are essentially different from one another. If the two apprehensions are independent, we miss that connecting link or relation between the premises and the conclusion which is called implication and is the basis of all inference.

The minor term in a syllogism need not be an individual, it may be a universal. Take the following example :

If all men are mortal,
and intellectuals are men,
then intellectuals are mortal.

Here 'intellectual' is not an individual but a species. How can the principle *Nota notae est notae rei ipsius* be applied here. For the sake of convenience a similar principle may be formulated: 'Whatever qualifies a genus qualifies the species contained within it'. The genus is a larger class containing the species or smaller class. As the class is to be understood extensionally, we have again to face the fallacy of *petitio principii*. If the genus and species are taken as universals or characters, then we have to explain and justify the process by which we *pass from* the apprehension of connection between two universals in the major and the apprehension of connection between two universals in the

minor *to* the apprehension of connection between two universals in the conclusion. Is there any connecting link between these apprehensions? As each is independent, it is difficult to find any such link. Moreover, as we have already stated, there is no possibility of perceiving a connection between universals. Nor is there any empirical ground for believing in the existence of universals. Therefore the Dictum in any form is unable to explain the principle of syllogistic necessity, i.e., the relation of implication on the basis of which inference is possible.

Aristotle seems to have thought that we are assured of the validity of the first figure by the fact that it conformed to the Dictum, this Dictum itself being a self-evident principle. He does not seem to have attempted to formulate a self-evident dictum for the other two figures. The validity of the second and third figures is guaranteed by the fact that they can be reduced to the first figure. But such a reduction is criticised by logicians and seems to be superfluous. The syllogisms in the second and third figures do not merely present under a disguise the reasoning of the first; they are independent types.

Let us take a syllogism in the seond figure :

(A) If all fishes breathe through gills,
and whales do not breathe through gills,
then a whale is not a fish.

It is supposed to be reduced to the following :

(B) If what breathes through gills is not a whale,
and a fish breathes through gills,
then a fish is not a whale;

from which the original conclusion that a whale is not a fish is recovered by conversion. It is clear that the two syllogisms (A) and (B) are different from one another. In the syllogism (A) we rely on something in whales which are the subject of our thought; in the syllogism (B) we rely on something in fish which becomes the subject of our thought, and it is by the process of conversion that whales again become the subject of our thought. Thus the movement of thought in the two syllogisms is different. One cannot be reduced to the other.

We may now consider the third figure. Take the following example :

> If no undergraduate may be a member of the executive-council of the university,
> and some undergraduates are electors to the executive-council of the university,
> then some electors to the executive-council of the university may not be its members.

To reduce it to the first figure, the minor premise would be changed to 'some electors to the executive-council of the university are undergraduates'. But then undergraduates are no longer the subject of our thought or instance which shows that a man may be entitled to vote without being entitled to be its member; the middle term now becomes a status in virtue of which certain voters cannot be the members. The point is that the movement of thought changes. Therefore the reduction of the third figure to the first figure is improper and superfluous.

Aristotle seems to have formulated the Dictum as a principle of all reasoning. Reasoning, according to him, is always syllogistic. Closely associated with this doctrine is the doctrine that all propositions are of the subject-predicate form. Where this form is not obvious, a proposition has to be restated. For example, the proposition 'Rāma killed Rāvaṇa' is to be restated in the form 'Rāma is a man who killed Rāvaṇa'. But is this restatement logically suitable? The proposition 'Rāma killed Rāvaṇa', asserts a relation between two terms 'Rāma' and 'Rāvaṇa'; it does not attribute a characteristic to a subject. We do not get rid of the relation by expressing it in the form 'Rāma is a man who killed Rāvaṇa'. Such a restatement is logically futile and practically absurd. The relation that holds between 'red' and 'this paper' in the proposition 'This paper is red' is quite different from the relation that holds between 'Rāma' and 'Rāvaṇa' in 'Rāma killed Rāvaṇa'. The traditional limitation to two constituents (subject and predicate) in a proposition and to one logical form was an undue simplification. In the relational proposition 'The headmaster teaches Sanskrit to his students', there are three terms and it cannot be expressed in the subject-predicate form by any restatement. A proposition may have any number of constituents and these constituents may be combined in various ways. The mode in which the constituents are combined determines the logical form of the proposition.

If every proposition is not of the subject-predicate form, it is a mistake to say that all reasoning is syllogistic whose fundamental principle is the *Dictum de omni et nullo.* There are arguments which cannot be expressed in the syllogistic form, though traditional logicians made various attempts to express such arguments in that form. But these attempts were logically futile and absurd. Consider the following examples :

1. If A is longer than B,
   and B is longer than C,
   then A is longer than C.
2. If A equals B,
   and B equals C,
   then A equals C.

These arguments are relational and not syllogistic. The Dictum, which presupposes syllogistic reasoning, cannot be the principle of their validity. Here the validity of implications depends upon the logical property of *transitivity* of the relations involved.

Now consider the following examples :

(A) If A is different from B,
and B is different from C,
then A is different from C.

(B) If A loves B,
and B loves C,
then A loves C.

These arguments are invalid. The reason is that the relations which they contain are non-transitive. The validity of an implication in relational arguments depends upon the property of transitivity of relations.

A word must here be said of the use of variables. The introduction of variables into logic is Aristotle's great invention. He presents the syllogism in variables in order to show that we get the conclusion not in consequence of the matter of the premises, but in consequence of their form. A variable is a symbol which denotes any one of a class of elements, and is, therefore, a mark of universality. Variables in a syllogism express that such a conclusion will follow always and for any value we choose. In modern formal logic expressions like 'for all A' or 'for all B', where A and B are variables, are called universal quantifiers.

It is usual to say that Aristotelian logic is formal. Here is a quotation from the *History of Philosophy* by Father Copleston : "The Aristotelian logic is often termed formal logic. Inasmuch as the logic of Aristotle is an analysis of the forms of thought—this is an apt characterization." But what is meant by the expression 'forms of thought'? "By form we mean what is the same in many individuals called *materially* defferent – the device, for example, on different coins struck from the same die, or the anatomical structure of different mammalian vertebrates, or the identical mode in which the law requires the different colleges of the University of Oxford to publish their accounts".[1] Thought is always about something. Pure thought is entire abstraction and is as good as nothingness. Things about which we think determine the forms of thoughts. Form and matter are inseparable. Differences in our thought arise from differences in that about which we think. We think in different ways about subjects of different kinds. A particular form of thought may be said to correspond to a particular kind of objects. For example, corresponding to the kind of objects called horses, there is the form of thought 'horseness'.

The major premise of a syllogism asserts a relation between two universal terms or classes which are expressed by variables. These variables belong to the form of the syllogism. Besides the number and disposition of the variables, the logical constants also belong to the form of the syllogism. Two of them, conjunctions 'and' and 'if-then', are auxiliary expressions. The remaining four constants, viz., 'to belong to all', 'to belong to none', 'to belong to some', 'to not-belong to some,' represent relations between universal terms or classes.

When it is said that Aristotelian logic is formal, the meaning is that in the Aristotelian syllogism the implication holds good for all values of the variables. But the values of each variable are fixed. We may substitute any concrete term denoting any member of a *particular* class for which the variable stands, but cannot substitute a term denoting another class. These classes and their relations determine the form and matter of a syllogism. It is formal to the extent it is concerned with *common characteristics* of the objects denoted by the terms; it is material to the extent these common

1. Joseph, *An introduction to Logic* (1925), p. 4.

characteristics are exhibited in *materially different* objects of the same class which is *different* from other classes.

Though Aristotelian logic is formal, it is not formalistic, whereas the logic of the Stoics is both formal and formalistic. Formalism is the doctrine according to which the same thought should always be expressed by means of exactly the same series of words ordered in exactly the same manner. It lays great emphasis on the precision of language. On the basis of its external form we can control the validity of an argument, without referring to the meaning of the terms used. For example :

If p, then q
p
therefore q.

Here we need not know the meanings of p and q. It suffices that the two ps contained in the premises have the same external form.

But no such magic seems to be involved in the Aristotelian logic, however much it may be so in the modern symbolic logic. Symbols are simply convenient means of expressing classes and their relations. The class is the meaning of the variable, and, as has already been said, it determines the form and matter of the syllogism. We cannot consider the validity of syllogistic implication without reference to the meanings of the variables which it contains. The essence of the syllogism depends not on words, but on their meanings and, therefore, we may exchange equivalent expressions, word for word, phrase for phrase.

Against this view, Stoics hold that if the words are replaced by their equivalents, the syllogism ceases to exist. Let us take an example from the logic of Stoics :

If α, then β,
but α,
therefore β.

If we replace 'If α, then β,' by its equivalent 'α entails β', we get —

α entails β,
but α,
therefore β.

But this, according to Stoics, is not a syllogism. It is only a valid rule of inference.

One further question about syllogism must be considered. It is usually said that Aristotelian syllogism is only formally valid. To call it valid is not to call either its premises or its conclusion true, but to say that *if* the premises are true, the conclusion is true. Does this mean that it has no reference to facts? Can we follow out syllogistic arguments with symbols, without knowing the facts to which they refer? Can there be a logic of consistency not related to the logic of facts or truth? To answer these questions, let us first understand the meaning of 'formal validity' or 'consistency'. To be consistent means that 'if we have admitted certain thoughts or facts, certain other thoughts or facts (said to be consistent with the former) must be admitted'. Consistency is not a matter of arbitrary symbolic convention, it is determined by the *nature of things*. Laws of thoughts are also the laws of things. But if we are working with empty symbols, how have we the nature of things before us? In all hypothetical reasoning facts are not actually present before us, but only *supposed* and there is a possibility of their actual existence. In a syllogism certain consequences follow in virtue of certain *supposed* facts. Therefore syllogistic implication may be said to depend upon an insight into the connection between the supposed facts expressed by the antecedent and the consequent. But in Aristotelian metaphysics facts are independent of one another. How then can there be an insight into the connection between them?

In knowing all implication there is a movement of thought, but there can be no movement of thought without a perception of connection between the objects of thought. In the inductive sciences we form hypotheses whose consequences we deduce, only to reject the hypotheses if their consequences differ from the observed facts. In deduction the premises and the conclusion both may be false, yet the knowledge of implication rests on perceiving connection between supposed facts. But again the same question arises. Is this perception or insight complete and reliable? Can there really be any connection between independent facts?

It is sometimes said that the major premise gives the rule which is taken on trust. Its essential nature consists in connecting a predicate with the middle term universally. According to Kant, the syllogism subsumed a cognition (i.e., a subject of knowledge) under the condition of a rule, and thus determined it by the predicate of the rule. These rules are the first principles upon which all know-

ledge must depend. But how do we acquire the knowledge of these first principles? It may be mentioned here that Plato worked out a theory of conception; a theory much criticised by Aristotle who never perhaps recognised how essential some such theory was to his own logical doctrine. Plato makes Socrates argue that for thought to be possible, we must have some apprehension of 'Ideas' or 'Forms' which are eternal, universal verities. Aristotle does not accept any such theory, nor does he replace it by a better one. In the celebrated passage[1] where he attacks the problem of how we acquire the knowledge of first principles, he rejects generalisation from experience as an explanation, and sees difficulty in an intutionist theory,[2] but he has no theory of his own to suggest. Thus the major premise remains a mere postulate for the sake of argument. Logic has nothing to say as to how we acquire the knowledge of the first principles. Knowledge is there before we start.

It is here that Kant criticises the traditional logic which treated conception as prior to judgment and inference; whereas according to Kant, judgment and inference are prior to conception. He clearly says that concepts are formed by means of a series of judgments. For the traditional logicians the acquisition of these concepts, which are the basis of all knowledge, remains a mere mystery. These concepts being universal and objective cannot be explained by an intuitionist theory because intuition is always private and individual, and, therefore, Kant's theory seems to be better. Another criticism by Kant against the traditional theory is that it regarded judgment and inference as analytic. According to the traditional logic, neither in judging nor in inferring do we add anything to what we knew before. A judgment or proposition is simply an analysis of a concept. In inference conclusion is merely an analysis of the premises. Kant's concern is to explain how synthetic a priori judgements are possible. Such judgements are universal and certain and add to our knowledge. Even before Kant, Hume had shown that thinking is synthetic, but the old logic had no account to give of synthetic thinking.

1. *Analytica Posteriora*, ii, 19.
2. According to Joseph, Aristotle seems to accept that thc truth of the major premise is known by intellectual insight. See Joseph, *An Introductian to Logic* (1925), Chap. XVIII, pp. 381 to 385. But such a view is not consistently expressed by Aristotle.

Among modern philosophers Descartes subscribes in essentials to the traditional doctrine that inference is deduction. On examining the method of geometry, he comes to the conclusion that new knowledge is deduced from simple first principles which are the foundations of all geometrical knowledge. These first principles or axioms are known to be absolutely true before any knowledge of their implications, and their truth does not in any way depend upon those implications, The central doctrine of Descartes is that the truth of these axioms is immediately known by natural light or *intuition*. In addition to the intuition of these first principles, we have also *intuitions of connections* between propositions, that is, the *intuitions of the implications* of these axioms. Thus Descartes seems to maintain that there are two kinds of intuitions : first the intuition of the truth of axioms; and, secondly, the intuition of their implications which is primarily an intuition of necessary connection, and secondarily an intuition of the truth of the implied propositions. It may be noted here that for Descartes thinking proceeds by intuition and not by analysis. Kant seems to be wrong in saying that Descartes treated thinking as analytic.

Descartes generally maintains that the intuition of the truth of axioms is of the same kind as that of their implications. This may be taken to mean that the intuition of the necessary connection between the axioms and the deduced propositions and the intuition of the truth of the deduced propositions, are one and the same thing. But this would make all hypothetical deductions true. Another difficulty is that if we allow the intuition of the truth of axioms and the intuition of their implications to be of the same kind, we have then denied their peculiarly axiomatic nature as distinct from the deduced propositions.

The main criticism against the intuitionist theory is : what is the guarantee of the possibility of such intuitions? Even if there is any, how can we rely upon it? Can it be communicated to others? Does it possess universal and objective character? Intuition being private and subjective cannot be said to possess universal and objective character, and, therefore, it cannot be the source of any true knowledge whose fundamental characteristics are universality and objectivity.

Against such an intuitionist theory, British Empiricists were mainly concerned with the formulation of an empiricist theory of knowledge. According to them, there are no a priori general

principles or axioms. All knowledge is derived from sense-experience. The following quotation will make the point clear : "Let us then suppose the mind to be, as we say, a white paper, void of all characters, without any ideas; how comes it to be furnished? Whence comes it by that vast store, which the busy and boundless fancy of man has painted on it with an almost endless variety? Whence has it all the materials of reason and knowledge? To this I answer in one word, From experience : in that all our knowledge is founded and from that it ultimately derives itself. Our observation, employed either about external sensible objects, or about the internal operations of our minds, perceived and reflected on by ourselves, is that which supplies our understanding with all the materials of thinking. These two are the fountains of knowledge, from whence all the ideas we have, or can naturally have, do spring."[1]

Though British empiricists explained all knowledge to be derived from sense-experience, the search for the basic principles of knowledge was not abandoned. Locke maintains that knowledge is due to the non-sensuous perception of agreement or disagreement between our ideas. Berkeley attacked physical science, but accepted the non-sensuous knowledge of spirit. Hume concentrates on the causal principle, but when he finds that it is not certain, he turns sceptic. It was left to Kant to show how synthetic a priori knowledge is possible.

The revolution in the logical theories of the modern philosophers consists in rejecting the syllogism in view of their argumentation of science. Scientific thinking consists in generalising from experience and this process is called induction. According to the traditional logic, thought consists in deducing conclusions from fixed first principles. But it does not tell how we acquire the knowledge of these principles. Science teaches that the so-called first principles are mere generalisations from experience. The statement 'S is P' is not immediate but mediate, depending upon the knowledge of the necessary connection. It must have been known by some such way as this – aS is P, bS is P, cS is P..., therefore, S is P. But such statements as expressed by the traditional logicians conceal the mediate process by which the man of science comes to apprehend them.

1. Locke, *An Essay concerning Human Understanding.*

No doubt science made an important contribution to logic by introducing induction as a method of knowledge, but it cannot be said that reasoning is never syllogistic. In everyday life we often reason syllogistically, and if our arguments are not explicitly in the syllogistic form, we have to put them in that form when conclusions are challenged. Certainly it is impossible to maintain that all reasoning is syllogistic.

While some critics reject syllogism on the ground that it involves a *petitio principii*, others try to defend it against such criticism. Those who defend point out that the question is whether the conclusion forms part of the evidence upon which the major premise is based. This would be the case if the major premise is obtained by *complete enumeration*, or by generalisation from a number of instances which include the minor term. But not all major premises are known by such generalisation. We can know them either by intuition or on the basis of the law of uniformity of nature. Therefore we cannot say that the conclusion forms the part of the evidence upon which the major premise is based. The major premise can be known without knowing the conclusion, that is, it is *epistemically independent*. *Constitutively* the premises *imply* the conclusion, but its converse is not true as implication is a non-symmetrical relation.

Mill considered that the major premise of a syllogism is obtained by generalisation from particular instances. For him the form of the inductive argument is :

> S_1 is P, S_2 is P, S_3 is P, and so on;
> therefore, every S is P.

According to him, the major premise of a syllogism is *not logically necessary*, but, being a memory-saving device, it is *psychologically useful*. It is a sort of memorandum, as he says of it, to which we snbsequently refer in order to save the trouble of repeating our observations.[1] But if the major premise is not logically necessary, inference in made non-syllogistic. Mill holds that the evidence for the major premise and for the conclusion is the same, therefore, we can directly pass from the observation of particular instances to the particular case asserted in the conclusion of the syllogism. Here he seems to be mistaken. The conclusion of

1. See *A System of Logic*, Bk. 11, Chap. iii, 4.

the syllogism is derived on the basis of the relation of implication holding between the premises and the conclusion, but it cannot be accepted that the premises asserting the particular instances imply that conclusion. The real problem is how far we are justified in making a general proposition (major premise) on the basis of a few instances. Inductive logicians justify such generalisation on the basis of their faith in the uniformity of nature. Whatever may be the justification for asserting the general proposition, it is only when it is stated that we can deduce a conclusion regarding a particular case.

We have already seen the difficulties in accepting the intuitionist theory for the justification of the major premise. Let us now see whether the law of uniformity of nature justifies it. The law of uniformity of nature is itself a generalisation from experience. We have experienced some uniformities in nature on the basis of which we infer that nature is uniform. But the uniformities which are said to be the empirical basis of our generalisation —the nature is uniform—are not really matters of direct experience. Every uniformity in which we believe is inferred inductively. So the law of uniformity of nature is itself the result of induction and to justify any induction or generalisation on its basis would be *petitio principii.* We may have *faith* in it and in inductive conclusions and such faith may be quite useful for practical purposes, but *no inductive conclusion is logically justified.*

As there is no justification for the major premise, the *Dictum* which tries to explain the syllogistic necessity in terms of it cannot be said to be valid.

There is another serious difficulty in explaining the syllogistic necessity. Terms, it is said, are independent of one another. They have no intrinsic or necessary connexion and do not modify each other. Every proposition states a relation between two independent terms. If there is no necessary relation between terms, there cannot also be any necessary relation between the premises and the conclusion in a syllogism, for they only state relations between terms. Moreover, the propositions in a syllogism in its inferential form express independent facts. In the absence of any necessary relation between facts, how can we necessarily pass from one to the other?

If the truth about the world is ultimately to be told in the statements of the form 'S is P', it requires that the world must

consist of independent substances which are the ultimate sujects of all predication. This seems to be the view of Aristotle. The ultimate subject of predication are the concrete individuals like Socrates or Krishṇa. Concrete individual things are called by Aristotle first substances. If it is asked what essentially an individual is, we will have to specify in our answer some kind of substance. The kind, e.g., man, gold etc., is called the second substance. All that is said of an individual, his essence (substance), quality, relation, activity etc., is in one of the categories. These categories or predicates differentiate one concrete individual from another.

At one time the man of science himself assumed some such pluralistic account of the universe. But the main influence of science has been to emphasise the unity of the world at the cost of its plurality. In the face of modern science, metaphysical (or idealistic) logicians also discarded the pluralistic account. They accepted the systematic or organic nature of the universe. It is not the individual but the Systematic Reality which is the ultimate subject of every judgment. According to metaphysical logicians, the nature of the universe cannot be represented by independent propositions of the form 'S is P' as understood by the traditional logicians. On the traditonal view the proposition 'S is P' simply connects the two terms, but does not assert any intrinsic relation between them. "The essence of inference is placed in attaching to a subject a predicate or succession of predicates, linked with it by connexions through predicates which are further used as subjects. A highly typical application of it is the sorites: A is B, B is C, C is D, therefore, A is D. In a syllogism understood after this ideal, the terms, it has been said, far from coalescing, rather tend to bid each other good-bye."[1] This is called the *Linea*r ideal of inference by Bosanquet. Against such a theory, metaphyiscal logicians developed the theory of *Absolute Systematic implication* which we shall discuss in chapter IV.

1. Bosanquet, *Implication and Linear Inference* (1920), pp. 21-22.

3

Linear Implication : Nyāya

After examining Aristotle's views, it would be proper to state and examine the Naiyāyika's views regarding inference and implication, for there are striking similarities between these ancient thinkers. Like Aristotle, the Nyāya metaphysics is a realistic pluralism and things are related to each other not internally but externally. Samvāya, which is an inseparable relation between some entities, is really an external one. It is a relation between a substance and a quality, a substance and an action, a genus and an individual, an eternal substance and its particularity, a whole and its parts. Due to his insistence on pluralism and the independence of terms, the Naiyāyika, like Aristotle, finds it difficult to explain the logical necessity involved in a syllogism. One main difference between the Aristotelian and Nyāya syllogisms may be noted at the outset. Aristotle formulated his syllogism as an implication, while the Nyāya syllogism is an inference. In implication the premises and the conclusion are only *stated*, whereas in inference they are *asserted*. Inference is based on implication and involves in addition certain psychological processes. Therefore the task before the Naiyāyika is wider and more complicated than that before Aristotle.

In Indian philosophy the word 'Anumāna' is used for 'inference'. Anumāna, as already stated in chapter I, literally means knowledge after something. It is mediate knowledge derived through the medium of some other knowledge. It depends upon the perception of a mark and the recollection of invariable concomitance. Anumāna is sometimes defined as knowledge which is preceded by perception.[1] Vātsyāyana holds that no inference can

1. *Nyāya-Sūtra*, 1.1.5.

follow in the absence of perception. First, there is the perception of a mark (linga), or probans (e.g., a smoke) in a subject (e.g., a hill). Secondly, there is the recollection of invariable concomitance of the mark with a predicate, probandum (Sādhya, e.g., a fire). Thirdly, there is the inference of the existence of the predicate (e.g., the fire) in the subject (Pakṣa, e.g., the hill). Some points of distinction between perceptual and inferenitial knowledge may be noted : (1) Perception is confined to present and near objects, while inference is related to past, future and remote objects also. (2) Inference depends upon the remembrance of a vyāpti, or a universal relation, which is not the case with perception. (3) Inference is mediate and perception is immediate knowledge of an object. Bhāsarvajña in his Nyāyasāra defines inference as the means of knowing a thing beyond the range of the senses through its 'inseparable connection with another thing' which lies within their range.

There are five members or avayavas in the Nyāya syllogism. The following is a typical Nyāya syllogism:

(1) This hill has fire (Pratijñā, or the proposition).
(2) Because it has smoke (Hetu, or the reason).
(3) Whatever has smoke has fire, e.g., an oven (Udāharaṇa, or the explanatory example).
(4) This hill has smoke which is invariably associated with fire (Upanaya, or the application).
(5) Therefore this hill has fire (Nigamana, or the conclusion).

The proposition (pratijñā) is the thesis to be established. It is the suggestion or the hypothesis which is set forth in the pratijñā. The suggestion fixes the problem and controls the inquiry. The proposition has two factors: (1) subject or minor term or pakṣa and (2) predicate or major term or sādhya. Pakṣa is what is observed and is generally an individual or a class about which we want to prove something. The sādhya is that character of the pakṣa or minor term which is not perceived by us and is to be inferred. The proposition should not be opposed to direct perception or the testimony of the scriptures.

The second member of the syllogism (hetu) states the presence of the middle term or liṅga in the minor term (pakṣadharmatā). The middle term is called the liṅga or sign because it serves to indicate that which we do not perceive. It is also called the hetu or

sādhana because it is the ground of our knowledge of the sādhya. There are five characteristics of the valid reason (hetu) or mark (liṅga): (1) existence in the subject (pakṣadharmatā, (2) existence in similar instances in which the predicate exists (sapakṣasattva), (3) non-existence in dissimilar instances (vipakṣasattva) in which the predicate does not exist, (4) uncontradictedness (abādhitaviṣayatva), that is, it must not establish such contradictory objects as the coolness of fire, and (5) uncounterbalancedness (asatpratipakṣatva), that is, the absence of counter-acting reasons leading to a contradictory conclusion.

The third member, udāharaṇa, is the universal proposition which states the invariable concomitance (vyāpti) between the mark (hetu) and the character inferred (sādhya) supported by an example (draṣṭānta). The mention of the example indicates that inference is both inductive and deductive. We generalise on the basis of particular instances, and then deduce new truths on its basis. The third member, according to Dr. Seal, "combines and harmonises Mill's view of the major premise as a brief memorandum of like instances already observed, fortified by a recommendation to extend its application to unobserved cases, with the Aristotelian view of it as a universal proposition which is the formal ground of the inference."[1] Examples may be positive (sādharmya) or negative (vaidharmya). A positive example is that in which the property to be proved (major term, e.g., fire) and the ground (middle term, e.g., smoke) are present, as in the kitchen. A negative example is that in which the property to be proved and the ground are both absent, as in the lake.

The fourth member, upanaya, is the application of the universal proposition to a particular instance. It may be affirmative (e.g., this hill is smoky) or negative (e.g., this hill is not smoky).

The fifth member is the conclusion drawn from the preceding members. What is tentatively put forth in the first member is proved in the conclusion.

Some ancient Naiyāyikas recognised the syllogism as consisting of ten members. In addition to the five given above, the following are included: (1) jijñāsā, or the desire to know the exact truth of the proposition, whether the hill is on fire in all its parts or in only some; (2) saṁśaya, or doubt about the reason, whether

1. *The Positive Sciences of the Ancient Hindus* (1958), p. 252.

after all that which we regard as smoke is only vapour; (3) śakyaprāpti, or the capacity of the inference to prove the existence of the predicate; (4) prayojana, or the end that will be served by drawing the conclusion; (5) saṁśayavyudāsa, or the removal of all doubts about the relation of the middle to the major and its presence in the minor. These five members are the psychological factors of reasoning. These are, according to later Naiyāyikas from Vātsyāyana downwards, unnecessary for proof, though they help to make our cognitions clear.

Some Indian logicians accept only the three-membered syllogism. It may be noted that the conclusion repeats the first member, proposition, and the fourth member is a restatement of the second. Buddhist logicians like Dignāga, Dharmkirti and Dharmottara severely criticise the five-membered syllogism and accept only its three members. Buddhist philosopher Nāgārjuna is sometimes given the credit of starting the three-membered syllogism The Mimāṁsakas and the Vedāntins also admit only the three membered syllogism. According to the Vedāntaparibhāṣā, we can have the first three or the last three members in the syllogism.

The Nyāya syllogism may be compared with the Aristotelian syllogism. The Nyāya syllogism has five and the Aristotelian syllogism three propositions. But this difference is nominal. Out of the five propositions of the Nyāya syllogism, two appear redundant and we may easily leave out either the first two, or the last two which are essentially the same. If we omit the first two, the last three correspond with the major premise, the minor premise and the conclusion respectively of the Aristotelian syllogism.

Apart from this nominal difference, there are certain real differences between the Nyāya and the Aristotelian syllogisms: (1) As we have already pointed out in the previous chapter, no syllogism was formulated by Aristotle as an inference, but they are all implications whose form is 'if...then...'.As against this, a Nyāya syllogism is an inference whose form is 'this...therefore...'. (2) The Nyāya syllogism is inductive-deductive and is concerned with material as well as formal truth, while the Aristotelian syllogism is deductive and concerned only with formal truth. The Nyāya regards induction and deduction as inseparably connected, as two aspects of the same process. Inference is neither from the

universal to the particular nor from the particular to the universal, but from the particular to the particular through the universal. The universal major premise of the Aristotelian syllogism is not shown to be the conclusion of induction, but the exemplification (udāharaṇa) or the universal major premise of the Nyāya syllogism is the conclusion of induction from known instance. (3) While in the Aristotelian syllogism the major and the minor stand apart in the premises though they are connected by the middle term with each other, in the Nyāya syllogism all the three terms (sādhya, pakṣa and hetu) stand synthesised in the upanaya. Due to these differences and due to the existence of syllogistic reasoning even in pre-Aristotelian works of the Hindus, we may say that Gautama's five-membered inference is not borrowed from Aristotle, as is some times held,[1] but has been an independent and natural growth. What has been possible in one country is possible in another also because the laws of reasoning are universal.

The Naiyāyikas give us three different classifications of inference. According to the first, inference is of two kinds, namely, Svārtha and Parārtha. The former is for oneself and in it we do not require the formal statement of the different members in language. It is a psychological process. The latter aims at demonstrating the truth of the conclusion to other people and has to be presented in language. This classification has in view the use or purpose which an inference serves.

According to the second classification, inference is of three kinds: (1) Pūrvavat (2) Śeṣavat and (3) Sāmānyatodṛṣṭa. This classification is based on the nature of the vyāpti, or universal relation between the middle and the major terms. A pūrvavat inference is the inference of an unperceived effect from a perceived cause, e.g., we infer future rain from dark clouds in the sky. Here the liṅga (middle terms) is related to the sādhya (major terms) as its cause. A śeṣavat inference is that in which we infer the unperceived cause

1. In his book *A History of Indian Logic* (1978, P. XV), Dr. Vidyābhūṣaṇa says, "It is not inconceivable that the knowledge of Aristotle's logic found its ways through Alexandria, Syria and other countries into Taxila" (an ancient centre of learning in India).
As against this it is sometimes made out that Aristotle was much influenced by the Hindu theory which was conveyed to him by Alexander who is reported to have had conversations with the logicians of India. But little positive evidence of direct influence is available.

from a perceived effect, e.g., we infer past rain from the swift muddy flood. Here the middle term is related to the major term as its effect. Both pūrvavat and śeṣavat inferences are based on the the uniformity of causation. Sāmānyatodṛṣṭa inference is, however, based on the non-causal uniform relation between the middle and the major terms. We infer that the sun moves because, like other moving objects, its position changes. Here inference depends on certain observed points of similarity between different objects of experience. Vātsyāyana gives other meanings also of these kinds of inference: (1) A pūrvavat inference is based on previous experience of invariable concomitance of two things. We infer an unperceived fire from a perceived smoke on the basis of uniform relation between the experiences in the past. (2) A śeṣavat inference is inference by elimination, in which the inferred character is the residium of a process of elimination which excludes other characters. We infer that sound must be a quality because it cannot be a substance or an activity or a relation and so on. (3) Sāmānyatodṛṣṭa inference is that in which we do not perceive the relation between the major and the middle terms, but find the middle to be similar to objects which are related to the major term. We infer the existence of the soul from the qualities of cognition, pleasure, pain, desire, aversion and volition because these qualities must, like other qualities, inhere in a substance. This is inference by analogy.

According to the third classification, inference is of the following three kinds: (1) Kevalānvayi; (2) Kevala-vyatireki; and (3) Anvaya-vyatireki. This classification is based upon the different methods of establishing vyāpti. In kevalānvayi inference the reason (hetu) has affirmative uniform relation with the presence of the predicate (sādhya). The middle and the major terms agree only in presence, there being no negative instance of their agreement in absence. Here the major premise is a universal affirmative proposition. The minor premise and the conclusion are also universal affirmative propositions. Hence the kevalānvayi inference is a syllogism of the first mood in the first figure, technically called Barbara in Western logic. The following is an example of this type:

All knowable objects are nameable
The pot is a knowable object
Therefore, the pot is nameable.

In Kevala-vyatireki inference the middle term is the differentia of the minor term and is negatively related to the major term. It is devoid of positive instances in which the middle and major terms may co-exist. There is vyāpti or a universal relation between the absence of the middle and the absence of the major term. In Kevala-vyatireki inference the major premise is a universal negative proposition; the minor prcmise, a universal affirmative; and the conclusion, a universal affirmative proposition. Although one of the premises is negative, the conclusion is affirmative, which is against the general syllogistic rules of Western Formal Logic. There is no valid mood corresponding to it in Western syllogism. The following is an example:

> What is not different from the other elements has no smell,
> The earth has smell,
> Therefore the earth is different from the other elements.

Symbolically, a Kevala-vyatireki inference can be put in the following form:

> No not-P is M,
> S is M,
> Therefore S is P.

In Anvaya-vyatireki ' inference the middle term is both positively and negatively related to the major term. The vyāpti is arrived at through the joint method of agreement in presence and in absence. The following is an example of Anvaya-vyatireki inference:

> All things which have smoke have fire,
> This hill has smoke,
> Therefore this hill has fire; and
> No non-fiery things have smoke,
> This hill has smoke,
> Therefore this hill is not non-fiery,
> i.e., This hill has fire.

We shall now discuss the logical and the psychological grounds of inference. We have already stated in chapter I that Johnson has distingulshed between the *constitutive* and *epistemic* conditions of

inference. It would be better to recapitulate these conditions. The constitutive conditions are independent of the thinker and refer to the *propositions* and the *relations* that hold between them. The epistemic conditions refer to what the thinker who is inferring *knows*. These two sets of conditions may be restated as follows:

Constitutive conditions: (i) *p* must be true; (ii) *p* must imply *q*.

Epistemic conditions: (i) *p* must be known to be true; (ii) *p* must be known to imply *q* without its being known that *q* is true.

These constitutive and epistemic conditions are the logical and psychological grounds of inference respectively.

Let us discuss how far the Nyāya syllogism fulfils these conditions. Are the premises true and imply the conclusion? To discuss this question we shall take only the last three propositions of the spllogism (i.e., Udāharaṇa, Upanaya and Nigamana) as the first two are superfluous and not necessary for inference. The Udāharaṇa states the invariable concomitance (i.e., vyāpti) between the middle and the major term supported by examples (dṛṣṭānta). Vyāpti is the nerve of inference. Vācaspati, Jayanta and Udayana define vyāpti as an invariable (nitya), unconditional (anaupādhika) or natural (svābhāvika) relation between the middle and the major terms. The *Vedānta-paribhāṣā* (Chap. II—Anumāna Pariccheda) defines[1] vyāpti as the co-existence of the major term with the middle term in all the loci in which the middle term may exist. According to the Navya-Nyāya, vyāpti is such a relation of co-existence between the middle and the major terms that the major is not a counter-entity to any negation abiding in the middle, i.e., it is none of those things which are absent in the middle term.[2] The word vyāpti etymologically means pervasion. It is thus a correlation between two facts of which one is pervaded vyāpya) and the other pervades (vyāpaka). The cases of fire pervade those of smoke. The fire is the vyāpaka and the smoke is the vyāpya. When the terms related are of unequal extension, such as smoke and fire, it is called Viṣamavyāpti. All smoky objects are fiery, but all fiery objects are not smoky. As distinguished from

1. Vyāptiścāśeṣasādhanāśrayāśritasādhyasāmānādhikarṇyarūpā.
2. Athavā hetumanniṣṭhavirahāpratiyoginā sādhyena hetoraikādhikaraṇyaṁ vyāptirucyate, (*Bhāṣā pariccheda*, 69).

this, when the terms related are of equal extension it is called samavyāpti or equipollent concomitance. There is samavyāpti between cause and effect. We may infer the cause from the effect and the effect from the cause. A samavyāpti is a universal proposition which distributes both the subject and the predicate, while a viṣamavyāpti is a universal proposition which distributes only the subject. Again, there are two kinds of vyāpti, viz., Anvaya-vyāpti and Vyatirekavyāpti. The former is the invariable concomitance of the presence of the middle term with the presence of the major term, while the latter is the invariable concomitance of the absence of the major term with the absence of the middle term. Vyāpti is known by the joint method of agreement in presence and agreement in absence based upon repeated observation.

Vyāpti is the unconditional (anaupādhika), invariable relation between the middle and the major terms. Smoke has unconditional relation to fire; whenever there is smoke there is fire. But fire has no unconditional relation to smoke; there may be fire where there is no smoke. The relation of fire to smoke depends upon the presence of wet fuel as a condition.

How is the universal proposition, all smoky objects are fiery, arrived at? How do we pass from the particular cases to the universal proposition? This question takes us to the problem of induction. The Naiyāyika speaks of three methods: induction by simple enumeration, indirect proof or tarka[1] and intuition or Sāmānya-lakṣaṇa-pratyakṣa to establish the universal proposition.

The Cārvākas, who are radical empiricists, contend that it is impossible to ascertain that smoke is invariably and universally attended by fire at all times. We cannot generalise on the basis of particulars. Perception is the only source of knowledge and it does not help us to establish a universal proposition. It is limited to present facts and cannot tell us anything about the past, distant and future. The invariable concomitance between smoke and fire in all times, in all places, and under all circumstances can never be known.

The Buddhists, however, contend that causality (tadutpatti) and essential identity (tādātmya) are the grounds of vyāpti. A universal

1. Really speaking, tarka is not an independent method but only supplements the method of simple enumeration.

relation can be ascertained, even without examining all possible cases, if the two terms are related either by causal relation or by identity of essence. The law of causality is a universal law. If two things are related as cause and effect, they are always and everywhere related to each other. It is self-contradictory to say that there may be an effect without its cause. The latter Buddhists introduced the method of Pañchakāraṇī to determine effectively whether the relation between two objects is causal or not. The five conditions determining the causal relation are (1) neither the cause nor the effect is perceived, (2) the cause is perceived, (3) the effect phenomenon is perceived in immediate succession, (4) the cause disappears, (5) the effect disappears in immediate succession. Similarly, identity in essence is another ground of establishing Vyāpti. There cannot be any essential identity between things that are absolutely different. By essential identity is meant the relation between two different things that co-exist in the same locus (Sāmānādhikaraṇyam). Thus there is an essential identity between a śiṁśapā (species) and a tree (genus). If it be ascertained in one case that śiṁśapā is in essence a tree, it can be universally affirmed that all śiṁśapās are trees.

The Nyāya criticises and refutes the Buddhist doctrine that causality and essential identity are the grounds of vyāpti. Jayanta says that, since according to the Buddhists all things are momentary, there cannot be any causal relation between them. Since a cause exists only for one moment, it cannot produce an effect in the next moment. There can be no causal relation between two series (Santāna) either, because existence consists in momentary causal efficiency and series is only imaginary. The so-called series consists of moments, and does not continue to exist for more than one moment; but it is imagined to exist for some time. Moreover, it happens that different causes often produce the same effect, and in such cases it is difficult to infer any particular cause from any single effect. The relation of essential identity cannot also be treated as the ground of inference. If the two things are identical, then both must be perceived at the same time, and to infer one from the other is meaningless. Moreover, Vyāpti is an inseparable relation between two different things. There cannot be any relation between identical things. Relation presupposes difference. Nyāya urges that neither the relation of cause and effect nor that of essential identity of genus and species exhausts the field of inference, and there are

other types of inference or universal relation which cannot be brought under either of them (e.g., the rise of the moon and the tide of the ocean).

The Nyāya tries to establish vyāpti on the basis of empirical induction by simple enumeration supported by tarka or hypothetical reasoning, whereas the Buddhists assumed a priori principles of causality and identity of essence. The Nyāya holds that by examining a number of positive and negative instances[1] of agreement in presence and agreement in absence between two things, we can establish a universal relation between them. Agreement in presence and agreement in absence both are necesary.[2] It is not enough that we observe fire where there is smoke; we should also observe that there is no smoke where there is no fire. Bhūyodarśana, or frequency of experience, without a single exception (Avyabhicharita Sāhacharya), helps us to establish a universal relation. The Udāharaṇa mentions an example along with the universal proposition. While an example may be sufficient to illustrate a rule, it cannot by itself establish a universal relation. A valid universal must be free from all conditions (nirupādhikaḥ) which are suspected by oneself or with which one is charged by one's opponent (samāropita).

The conclusion arrived at by the method of simple enumeration may be confirmed by tarka or a hypothetical reasoning which *indirectly proves* its validity. Tarka by itself cannot give us a universal proposition. Tarka consists in showing that a denial of the universal relation leads to contradictions. If the general proposition, where there is smoke there is fire, is not valid, then its contradictory that "sometimes smoke is not accompanied by fire" must be true. But we cannot prove that sometimes smoke is not accompanied by fire. Hence the universal relation is confirmed. Tarka only strengthens the conclusion arrived at by the empirical method of induction. A universal proposition is strengthened by tarka, but it cannot be said to be absolutely certain. Enumerative universals are only probable. Therefore, the Nyāya tries to explain the certainty of universal proposition on the basis of Sāmānyalakṣaṇa pratyakṣa, the non-sensuous perception of the universal. In perceiving particular smokes, we perceive

1. For Vedānta negative instances are not necessary to establish vyāpti.
2. Sāhacharyajñāna and Vyabhichārajñānaviraha, *Tarka Saṅgraha*.

the universal smokeness inhering in them. The universal proposition 'all smoky objects are fiery' is given by a non-sensuous perception of smokeness as related to fireness. The validity of a universal proposition ultimately depends on the discovery of relation between common essences of universal characters. The connotations of smoke and fire are related in the vyāpti as the vyāpya and the vyāpaka. On the basis of a single instance we can obtain a universal connection. A multiplicity of instances is necessary only because the relation is not clearly differentiated in a single case.

Against this view it may be noted that there is no empirical basis for justifying the super-sensuous perception of universals. Even if it is accepted that whenever we perceive the particular we also perceive the universal inhering in it, there seems to be no justification for believing that we perceive the necessary relation between universals. If we perceive the latter, what is the necessity and use of repeating instances on which the Nyāya puts so much emphasis? As the universal relation cannot be conclusively and logically established on the basis of the empirical method of induction, the Nyāya has mixed its empiricism with the a priori perception of universals.

The knowledge of vyāpti alone is not sufficient for inference. The second condition necessary for inference is Pakṣadharmatā or the relation between the minor and the middle terms. In inference the minor term becomes related to the major through its relation to the middle terms. In every inference we want to establish something about the minor term (Pakṣa) on the basis of vyāpti. The knowledge of Pakṣadharmatā as qualified by vyāpti is called Parāmarśa. And inference is defined as knowledge arising through Parāmarśa.[1] Some Naiyāyikas hold that the knowledge of vyāpti, as that between smoke and fire, generates, on being reproduced by the perception of smoke on the hill, the knowledge : "The hill has smoke which is always associated with fire". Thus the function (vyāpāra) of the knowledge of vyāpti is to correlate the three terms S-M-P.[2] This function, by which the knowledge of vyāpti makes an inference possible, is technically called by the

1. Parāmarśajanyaṁ Jñānamanumitiḥ.
2. S, P and M symbolize the minor, the major and the middle terms respectively.

Naiyāyikas tṛtīya liṅgaparāmarśa, i.e., a consideration of the middle term for the third time. The middle term, e.g., smoke, is known for the first time when it is observed in the hearth, i.e., when we acquire the knowledge of its invariable relation with the major term, fire (vyāpti); it is known for the second time in relation to the minor term, e.g. the hill; and it is known for the third time when it correlates the three terms, as when we say 'the hill contains smoke which is always related to fire' (vahni-vyāpya-dhūmavān-ayaṁ parvataḥ). Another name of this function is 'vyāpti-viśiṣṭa-pakṣadharmatā-jñānam' which means knowledge of the character of the minor term as qualified by the knowledge that the character is also universally related to the major term. It is through liṅga-parāmarśa or knowledge of the middle term as universally related to the major and as characterizing the minor, that the knowledge of the vyāpti leads to the conclusion. Hence liṅga-parāmarśa is the immediate cause of inference. On this analysis inference involves the following steps :

1. M—P, the knowledge of vyāpti or the universal relation between the middle and the major terms.

2. S—M, the perception of the minor term as characterized by the middle (pakṣa-dharmatā-jñānam).

3. M—P, the recollection of vyāpti and its reproduction in the form of a conscious judgment.

4. S—M—P, the correlation or synthesis of the three terms tṛtīya-liṅga-parāmarśa, or vyāpti-viśiṣṭa-pakṣa-dharmatā-jñānam).

5. S—P, the conclusion relating the minor term with the major.

But the Advaita view regarding the function (vyāpāra) of the knowledge of vyāpti is different from that of Nyāya. The Advaitins hold that the knowledge of vyāpti is revived in our mind when we see the liṅga or the middle term as related to the pakṣa or the minor term, and this at once leads to the conclusion. There is no synthetic judgement (S—M—P) correlating the three terms. The following is the psychological analysis of the inferential process by the Advaitins :[1]

1. Liṅga-parāmarśa is not a necessary condition of inference, according to the Advaitins.

1. M—P, the knowledge of vyāpti or the universal relation between the middle and major terms.

2. S—M, the perception of the minor term as characterized by the middle (pakṣa-dharmatā-jñānam).

3. A revival of the impression of vyāpti previously acquired, without any conscious judgment.

4. S—P, the conclusion that the minor term is related to the major term.

Now we can clearly understand (using Johnson's terms) the *constitutive* and the *epistemic* conditions of inference as held by the Naiyāyikas and the Advaitins. According to the Naiyāyikas, the constitutive conditions of inference are : (1) the proposition 'M→P' (vyāpti) is true, (2) the proposition 'S—M' (pakṣa-dharmatā) is true, and (3) the proposition " 'S—M' as qualified by 'M—P' (parāmarśa) implies 'S—P' " is true.

The epistemic conditions of inference are :

(i). The proposition M—P is known to be true.

(ii) The proposition S—M is known to be true.

(iii) The proposition " 'S—M—P' implies 'S—P' " is known to be true.

For the Advaitins the above constitutive and epistemic conditions no. (3) and (iii) respectively are not necessary for inference. The knowledge of (i) and (ii) directly produces the knowledge of the conclusion. But the constitutive condition no. (3) is there whether we express it or not.

As the Naiyāyikas have formulated their syllogism as an inference, the main question before us is how far they are justified in asserting the proposition M—P (vyāpti). As we have already mentioned, they speak of two methods of arriving at universal propositions : (1) simple enumeration supported by tarka, indirect proof or hypothetical reasoning, and (2) non-sensuous intuition or sāmānya-lakṣaṇa pratyakṣa. In simple enumeration we pass from *some observed* cases of the relation between two things to a statement about their relation in all cases. From the premise 'all observed As are Bs' we infer the conclusion 'all As are Bs'. Since all observed As are not all As, the inferencc is not logically valid.

In passing from a premise about some As to a conclusion about all As there is an illicit distribution of A. It violates the general rule of inference that we must not go beyond the evidence. Our sensory powers being limited we cannot perceive instances of all times and places. A single contradictory instance may appear which disproves the conclusion. An interesting example of simple enumeration is the universal proposition 'all swans are white'. This conclusion was derived by many people from their observation of swans in Europe only. On the discovery of Australia black swans were discovered and the universal proposition 'all swans are white' was disproved. Hence the method of simple enumeration cannot conclusively establish vyāpti or a universal proposition. Simple enumeration is simply a process of counting instances and it does not involve any analysis or necessary connection between facts. For this reason Bacon and Mill seem to doubt the validity of simple enumeration as a method of establishing universal propositions conclusively. Thus Bacon says : "The induction which proceeds by simple enumeration is childish; its conclusions are precarious, and exposed to peril from a contradictory instance; and generally it decides on too small a number of facts, and on those only which are at hand."[1] Mill says : "Popular notions are usually founded on induction by simple enumeration; in science it carries us but a little way. We are forced to begin with it; we must often rely on it provisionally, in the absence of means of a more searching investigation. But, for the accurate study of nature, we require a surer and more potent instrument."[2]

Perhaps knowing that the method of simple enumeration cannot conclusively establish a universal proposition, the Naiyāyika has propounded the method of sāmānya-lakṣaṇa pratyakṣa as a surer instrument. The Nyāya view assumes that universals are categories of reality and universal relations are real and require only to be intuited. But do we ever perceive universals in the same way as we perceive other things? The immediate reply is 'no'. I perceive a particular cow and not the universal 'cowness'. We never have the non-sensuous perception of 'cowness' associated with the sensuous perception of a cow. The hypothesis of a non-sensuous knowledge of the universal existing in a particular object

1. *Novum Organum*, Bk. I, Aph. 105.
2. *A System of Logic*, Bk. III, Chap. III, * 2

is not necessary at all to explain the knowledge of that object as belonging to a particular class or genus. We determine the class of an object on the basis of its characteristics which we perceive by our senses. As there is no perception of universals, there cannot be any perception of the relation between them.

The Nyāya seems to hold that from the non-sensuous intuition of M—P and the sensuous intuition of S—M, we pass to the intuition of S—M—P (liṅga-parāmarśa) which gives rise to the intuition of the conclusion S—P. But as we have already stated, there is no justification for believing the non-sensuous intuition of M—P. Moreover, even if it is accepted, it is independent of the sensuous intuition of S—M. There is no connecting link between them. Then how can they be synthesised and give rise to the intuition of S—M—P. Unless we admit some connecting link between them, the movement of thought to S—M—P and then to S—P cannot be explained.

It should be noted that the terms M and P in the major premise express universals, while the terms S and M in the minor premise express particulars.[1] To synthesise the two premises as S—M—P we must first determine the relation beween the universal and the particular.

The Nyāya holds that universals have objective existence and are not conceptual constructs. It does not obviously agree with the Buddhist view that the gêneral notion is but a name. A universal is one (ekam), eternal (nityam), residing in many (anekānugatam). It is present in all objects of its class (svaviṣaya-sarvagatam) with an identical nature (abhinnātmakam) and is the cause of the notion of concordance (anuvṛttipratyayakāraṇam). There is the genus of cow (gotva) in many cows. The genus is one. It inheres in many individuals. The following are the characteristics of a universal: (1) It inheres in all its proper individuals. The genus of cow exists in all individual cows. It does not exist in other individuals, sheep, goats, etc. (2) The universal exists in all its proper individuals with an identical nature. It does not differ in its nature in different individuals. It is their common and essential nature. (3) The universal is the cause of the concept of common character in many individuals. A concept is produced by a uni-

1. Here by 'particular' we mean particular things like cow, Ramesh, fire in the kitchen etc., and not the category-Viśeṣa.

versal which is its objective basis. A universal is the class-essence existing in many similar individuals. (4) The universal is eternal. Individuals, which are its substrates, are produced and destroyed. But a universal is neither produced nor destroyed. Individual cows are produced and destroyed but cowness exists for ever. (5) A universal is different from the individuals which are its substrates. If a universal were not different from its substrates it would be produced and destroyed with them. But it is not so. The universal is eternal while its substrates are temporary.

Praśastapāda gives to sāmānya a reality independent of individual objects. While Kaṇāda insisted more on the inseparable relation between the universal and the individual, Praśastapāda emphasises the eternal nature of the universals. His view is that while the individuals are destroyed, the universals subsist even in the state of pralaya, or the destruction of the world. At the time of creation universals enter into the individuals and make for themselves temporary manifestations. Praśastapāda's view seems to be akin to Plato's idealism, according to which sensible objects participate in the universal forms of Ideas which are eternal and self-subsistent. All the objections urged against Plato's view may apply here also. A universal being one and eternal cannot be divided. How without division or multiplication can a universal participate or exist in many individuals? If to explain the common characteristics between two men, we postulate the existence of a universal man, then we require a still higher universal, the so-called third man, to connect the first universal with its proper individuals and so on ad infinitum.

Now we come to the main question, how far is the Nyāya justified in synthesising the two premises S—M and M—P as S—M—P? We have said that the terms S and M in the minor premise express particulars, while the terms M and P in the major premise express universals. How do we get the proposition M—P, i.e., 'wherever there is smoke there is fire?' The Nyāya holds that while perceiving one smoke as related to fire, we perceive all smokes through the universal smokeness as so related. But as the Nyāya cannot successfully explain the relation between the universal and its substrate, the particular, the non-sensuous perception (sāmanyalakṣaṇa) of the universal together with the sensuous perception of the particular cannot be admitted. I perceive a smoke, but not smokeness. Perhaps realising that the

method of simple enumeration cannot establish a universal proposition, the Nyāya has postulated the occurrence of sāmānya-lakṣaṇa pratyakṣa. No doubt the method of simple enumeration cannot logically establish a universal proposition, but it does give it a certainty of a very high degree which is useful for practical purposes. The hypothesis of the non-sensuous perception of the universal together with the sensuous perception of the particular cannot be sustained by facts or reasoning. If the proposition M – P (whenever there is smoke, there is fire) is not established, its synthesis with the proposition S—M (this hill is smoky) as S—M—P (this hill is smoky which is invariably associated with fire) is not valid and the conclusion S—P (this hill is fiery) is not logically established.

Though the Nyāya could not succeed in establishing the logical certainty of the conclusion in syllogism on the basis of Sāmānya-Lakṣaṇa Pratyakṣa, it does give to the conclusion a *probability* of a high degree on the basis of induction by simple enumeration. The Nyāya syllogism is better than that of Aristotle because its conclusion being highly probable is good and useful for practical purposes. From the practical point of view we are not very much interested in the formal or logical certainty of the conclusion. It is the high probability of the conclusion which makes it the guide and rule of our practical life. The Aristotelian syllogism, being purely deductive, was formulated by its exponent as an *implication* while the Nyāya syllogism, which is inductive-deductive, is an *inference* based on *evidence*. The Nyāya regards induction and deduction as inseparable processes of thought. Inference is neither from the particular to the universal nor from the universal to the particular, but from the particular to the particular through the universal. Though the Nyāya could not successfully explain and cstablish the relation of implication between the premises and the conclusion from a strictly logical or formal point of view, the premises support or are the evidence for the conclusion from the empirical or practical point of view. The conclusion is *not logically* necessary but *empirically certain* and there is sufficient reason to rely on it unless it is contradicted by any future experience.

No pluralistic system which holds the independence of things can successfully establish a necessary relation, between the different terms of a syllogism, on the basis of which alone the conclusion logically (necessarily) follows from the premises. The relation of

inherence is pointing towards some organic unity in the universe, but due to its thorough pluralism, the Nyāya could not make inherence a purely internal relation. Like the Aristotelian logic, the terms in the Nyāya logic are connected only externally. Instead of coalescing, they are busy in keeping their own independence. They are like different links of a chain which are connected by a *tour de force*. Whether we proceed from the links to the chain (induction), or from the chain to the links (deduction), or from the links to the links through the chain (induction-deduction), the relation between different links remains the same and may be termed linear. In spite of some differences both Aristotle and Naiyāyika recognise the independence of terms and independence of propositions in a syllogism. Hence the nature of implication in their logic is the same and may be called *linear*.

4

Absolute Systematic Implication

Idealistic logicians—Bradley and Bosanquet—criticised the traditional theory of inference and developed the theory of Absolute Systematic Implication on which all inference is based. What makes the traditional inference linear is respect for the independence of the terms. "Each term must preserve its independent being, as if enclosed in a bracket, and can only react on others by indicating the consequence of its presence or absence in respect of the presence or absence of the others."[1] Terms do not modify each other. Each preserves its own identity and essential nature. When a predicate is attached to a subject there is no bearing of the import of one upon the other. The terms are independent units and do not affect each other. They merely co-exist. But such independent terms may be said to have no relation with each other as mere co-existence is not a relation at all. To be related means to affect each other, to modify each other. Without a relation difference has no meaning. Plurality of terms depends on relation, and without that relation, they are not distinct. So metaphysical (or idealistic) logicians laid stress on the unity of terms. According to them, a judgement is not an assertion of the co-existence of two independent terms, but it is an assertion of their unity. Even before them Kant had made it clear that mere scattered independent sense-experiences cannot produce knowledge. They require the forms of understanding and the transcendental unity of apperception to become knowledge. But whereas for Kant it is ultimately the transcendental unity of apperception, for metaphysical logicians this unity is intrinsic to the systematic nature of reality.

1. Bosanquet, *Implication and Linear Infcrence*, p. 25.

According to metaphysical logicians, the whole judgemnt is a single content. It does not assert a relation between two mental states, but is itself the content of a single though continuous mental state. Terms are not themselves unities, but derive their unity from the judgment; judgment as a whole is the unit. In the judgment 'Fire causes heat', 'fire' and 'heat' are not two contents, but 'fire—causing—heat' is itself the single content. But if the whole judgment is a single content, what is the distinction between the subject and predicate ? The distinction is within the same content. It is the analysis and synthesis of elements in the same thing. Every judgment is the connection of parts in a whole. The explicit subject marks the place at which, or the conditions under which, the Reality accepts the predicate. The subject is concrete, and the predicate abstract; the subject real, and the predicate ideal. But the ultimate subject of every judgment is the Reality, and, therefore, every judgment can be thrown into a form in which the undefined Reality is the subject, and the whole judgment is the predicate. The judgment 'Mr. Desai is a great statesman' may be put in the form, 'The Reality is such that Mr. Desai is a great statesman'.

The characteristic mark of a judgment is that it is essentially bound to other judgments. The whole idealistic theory of implication and inference is based on this that when we make one judgment we are really asserting the whole system of judgment. Thinking is essentially systematic, and in achieving system it is not reproducing or copying an intuited unity, but producing a unity of its own. It constructs the system in accordance with a rule of its own. Bosanquet mentions[1] three properties of a judgment. It is (a) necessary, (b) universal, and (c) constructive.

A judgment is necessary or objective because it expresses what we are obliged to think. As the aim of thinking is always to achieve a system in accordance with certain laws, we are forced to think in a certain way. Given a certain context of ideas and perceptions, the judgment necessarily follows. This necessity is constantly expressed by such phrases as 'I am forced to believe'; The judgment is inevitable'; I am unable to think otherwise'; I am unable to resist the conclusion'. If a certain conclusion is modified, we will have to modify the whole system of knowledge.

1. *The Essentials of Logic*, 1948, p. 23.

The second property of a judgment is universality. Universality of a judgment is the corollary of its being necessary. My thought is not a mere momentary psychical state of my mind, but expresses something which all are obliged to think. Given the same materials, it it obligatory for every other rational being to think in the same way. The laws in accordance with which my thinking moves are not the laws of my thinking, but of *all* thinking. It is in this sense that every thought is objective and universal. The world which thought constructs is more than my psychical state. It is a world having certain laws which are the same for all. My judgment must be consistent not only with my own judgments but with the judgments of all.

The third property of a judgment is constructiveness. Every judgment is constructive because it is an affirmation in which our present sense-perception or data is amplified by an ideal interpretation. The process of interpretation is continuous and gives rise to a system. It may be admitted that the distinction between the present sense-perception or data and its interpretation is only relative. We can find no data unqualified by interpretation, that is, by judgment. To take an example, 'That is an idol' is a judgment and not mere sense-perception. It is very difficult to separate mere visual perception from the interpretation that 'That is an idol'. 'That' and'what' of a thing are inseparable.

If the mind *constructs* or produces a system in accordance with certain laws, and if it *necessarily* does so, then parts *cannot be* just *given* as they are to appear in the system. If the mind is to produce a system, it must be granted some freedom to modify the parts. This means that the terms of the judgment cannot represent realities directly apprehended; they cannot be ideas as given; they must be modifiable. If the terms are not modifiable, it might be possible to build a heap or aggregate, but it would be impossible to make them into a system or unity. System-building requires that every part must fit into the system, and if any part does not fit, it has to be made fit. This means that every term is slightly modified to made it fit. If thinking really makes a system, it must be granted a certain amount of freedom with the given which must be susceptible of such modification as is necessary to make it fit into the system.

Terms are not only modifiable but take their meaning from the judgment. If thinking is essentially systematic, what deter-

mines the meaning of the terms is the requirement of the system. When we say 'The sum total of energy is conserved', 'There are five energies, light, heat, sound, electricity and magnetism', 'Heat energy can be transformed into light energy', our meaning of energy is not always the same. Whenever a physicist makes a fresh statement about energy, his notion of energy is not the same again; it is developed and modified. He does not modify his notion of energy and then make the judgments, but makes the judgment first, and then determines the change in the notion of energy brought about by the change in the judgment.

Thus we have to abandon the axiom of the traditional logic that before we make a significant judgment we must know the meaning of terms. On the traditional view concepts are prior to reasoning. They are rigid and stable. Their meaning is determined before reasoning begins and remains the same throughout. In the face of developments in science the Idealistic logic abandoned this view.

In asserting a judgment we are really asserting the whole system of reality, therefore, the terms of a judgment must have their meanings fixed by the requirement of the system. If my assertion is wrong at any point, it is wrong at all points. The falsity of any statement 'A is B' consists in this that there is nothing quite corresponding to either A or B in the real system, and therefore the statement is false all along the line. The truth or falsity of any statement affects the truth or falsity of the whole system. The general formula to express the necessity of a valid statement is 'This or nothing'. If a true statement is not accepted, our province of truth as a whole would be taken from us. When a change is made in any term, it requires a change in the whole system.

Bradley speaks of ideas or terms as symbols in a judgment. These ideas are not received in direct apprehension of an independent reality, but are made by the mind. Yet they have a value as symbols because they can be manipulated by the mind in accordance with certain rules and thus express the reality which is necessary, objective and universal. Their meaning is determined by the whole system of reality which they produce. If an idea or term does not fit into the system, it must be discarded, and new ones must take its place. Judgment is the act by which we at once use and modify these ideas or symbols to make a system.

Let us now consider the idealistic theory of implication and

inference which is merely a corollary of the theory that thinking is essentially systematic. Inference is simply the working out, according to rules of the implications, of judgments which form a system. In each case of inference we are concerned with two systems: (1) the special given system (or complex) which is only a partial system and whose consequences we desire to draw, and (2) the total system of reality. Though our conclusion is based on the special given system, yet it refers to the whole system of reality. The special given systems are only the parts of the whole system of reality, therefore, the special conclusion is really a conclusion from the whole system of reality. The conclusion is false unless the whole system of reality confirms its truth. It is the individual system which, considered together with the whole system of reality, gives rise to special conclusion.

Bosanquet uses the expression 'true wholes' or 'true universals' for these special systems. These systems are true wholes in the sense that each part in the system responds to every other; modification in any one makes a modification in every other. Though each element within a system is unique and peculiar, yet it is related to every other. There is some unifying thread which runs through every part. To take an example, if a side of an equilateral triangle is lengthened, the opposite angle is enlarged. Change the notion of energy, and ultimately the whole physics must be modified. Alter the value of any combination in the numerical system, and ultimately the whole system must be altered.

Now what is the relation between different elements within a system? The relation is best expressed by the word 'implication'. Using the word 'term' for 'elements', Bosanquet explains the nature of implication as follows: "This is the general name for the relation which exists between one term or relation within a universal or connected system of terms and relations, and the others so far as their respective modifications afford a clue to one another"[1]..........."Implication as such is a relation of content to as a whole reality".[2] "...... It is the connection between terms or sets of terms in virtue of a common nature which binds them into parts within a continuous system such that you can tell from one part of it or more modestly, that there are parts of it from

1. *Implication and Linear Inference*, p. 9.
2. *Ibid.*, p. 12

which you can tell what the other parts of it, or some of them, are and how they are behaving.'"[1]

The essential characteristic of implication is that it is *necessary*. When we say that a certain conclusion follows from given premises, we mean that it necessarily follows. If A is north of B and B is north of C, then A *must be* north of C. This necessity is best expressed in the formula "This or nothing". There is a given complex which together with the whole system of reality implies a certain result, and this result must be admitted, so long as we affirm the whole reality. We can deny the implication only by denying the whole system of reality. The result is inevitable and is forced upon us by the nature of reality. If we annul the implication, we must modify our whole view of reality, and in such cases between 'this' and 'nothing' we have chosen 'nothing'. Either the implication stands or the whole system of reality falls.

But why is it so certain that there is a necessary element in inference or thinking? And if there is any such element, what is its nature? These are the important questions of philosophy. Hume denied any element of necessity and universality in knowledge. Thinking is controlled only by the laws of association. But these laws are not the laws of inference. There does not lie any necessity in the sequence of the states of mind. Psychological sequence is not what we mean by logical necessity. So Hume's philosophy leads to scepticism.

His scepticism shattered the foundations of all necessary knowledge. It was left to Kant to prove that there is an element of necessity and universality in knowledge. The necessity and universality in knowledge do not come from sense-experience but from the categories of understanding. These categories which are the forms or moulds of all knowledge are a priori and are derived from the transcendental unity of pure apperception which gives unity to all experience.

Idealistic logicians, like Kant, maintain that there are necessary and universal laws of thinking, but they give a different account of these laws. Kant's necessary and universal laws are mere static forms or moulds of knowledge to which the objects must conform, but they do not tell us anything about the reality. For the idealistic logicians these laws are the dynamic aspects of

1. *Implication and Linear Inference*, p. 14

reality and it is in accordance with these laws that thought constructs the whole system of reality.

Hume tried to prove that the so-called necessity in knowledge cannot be placed in the sequence of the states of mind. But then does it lie in the facts? Idealistic logicians repudiate any such attempt because in their view there are no independent ready-made facts to be apprehended or intuited by the mind. In the hypothetical argument, 'If A is B and B is C, then A is C', there is the necessity independently of whether the statement 'A is B' states a fact, or even of whether there are such things at all as A or B or C, outside and independent of my judgment.

Idealistic logic explains this necessity on the basis of the coherence theory of truth which avoids the distinction between formal and real validity. We often distinguish between the case where a certain conclusion necessarily follows from the given premises, and the case where no premises are given to draw the conclusion, but its validity is proved *as a matter of fact*. The former is said to be *formally* valid, and the latter *materially* valid. It is sometimes maintained that formal validity is independent of material validity, that an argument may have absolute formal validity without having material validity. A theory in which the whole argument is strictly mathematical has absolute formal validity, though it may be false as a matter of fact. It is internally coherent, and if any single proposition is true they are all true, if any is false they are all false. We cannot doubt the internal coherence of Euclidean geometry, though we may doubt its material truth.

Even if thinking had absolute formal validity, could it have material validity? Can it give us any knowledge of external reality? Kant held that the objects of thought have to conform to the laws of thought, and thus we get universal and necessary knowledge of objects. But ultimately these objects are mere appearances or phenomena. Our thought cannot go beyond these appearances. It cannot grasp the Noumena. Reality falls outside the domain of thought and can never be known.

Idealistic logic deals with the problem in a different way. At the outset it denies the existence of any independent ready-made reality to be known by immediate apprehension. As we have already seen, it criticises the doctrine of immediate apprehension root-and-branch. On its view mind does not copy or intuit any

reality but produces reality. There are seriovs objections to the doctrine of immediate apprehension. Can a unitary coherent theory as a whole be known to be true or false by a single immediate apprehension? If this is possible, then immediate apprehension is nothing more or less than magic. If a series of apprehensions is required, then the truth or falsity cannot be said to be known immediately, but mediately. Further, how do we know that a certain theory is partly true and partly false? How are we to know that a certain theory is neither true nor false, because it is framed in terms which are neither true nor false? If immediate apprehension can give more than truth *or* falsity, its character as immediate is certainly lost.

There is diametrical opposition between the view that thought produces the reality as a system and the view that brute facts are given. Idealistic logic accepts the first view. The coherence theory of truth is a corollary of this view. According to this theory,the test of thinking is not externāl but internal, a theory is true not because it corresponds to the external facts but because it is internally coherent. Coherence and truth are not two things but one and the same thing. Truth is coherence and coherence is truth. Thus the distinction between formal and material validity is abandoned. On the coherence theory there are no independent judgments which are immediately recognized to be true. Each judgment is essentially a part of a system of judgments and the whole system stands or falls. Advance of science proves that those theories which are more coherent systems are accepted and the less coherent are rejected. Copernican theory is accepted because it is more coherent than that of Ptolemy. Change from one system to another is not a random substitution, but a real development towards complete coherence or absolute truth.

On the coherence theory of truth, all implication being based on the coherence of judgments necessarily gives us truth. What is implied by the premises is necessarily true. If any inference is rejected as false, it is only because it does not cohere with the whole system of reality.

Even the acceptance of brute facts depends on coherence or implication within a system. We accept a fact because our whole system of knowledge would be shattered if we did not accept it. Our acceptance of facts as facts depends upon our systematic theory. Propositions of facts presume necessary coherence. If

we accept that the excision of the thyroid gland must dull the intelligence, it is because it coheres with other propositions about the human body. The decision to accept a given complex as a fact depends upon its being an implication of some further complex which is already established as a fact.

It is obvious that all implication is in strict logical character a priori. Its a priori character lies simply in the absence of complication and confusion. It is clear and distinct and its truth can be known by intuition. Of course actual certainty is a matter of degree depending on the distinctness of any element in the given system together with the whole system. But the nature of implication is always the same. If certain elements of a system are given, we can tell by intuition what the other element must be.

But how are we to know the implications which are a priori truths ? The answer is '*by insight or intuition*' of the type that two straight lines cannot enclose a circle. The intuition is different from the intuition or immediate apprehension of independent facts as maintained by the traditional logicians. Bosanquet says, "The a priori character of an intuition depends on the distinctness of the whole which is its foundation, together with the depth or comprehensiveness which determines the degree of inherence with which that whole is rooted in the reality. It is subordinated by the subordination of that whole to a wider or more truly real one, but is not destroyed. Under its own conditions or reservations it maintains its relative truth, whereas ultimate truth could belong only to intuitions founded upon the ultimate system, such as that of the unreality of the self-contradictory, which is necessary to the recognition of a whole in any sense. Obviously, the character of the a priori can be brought by modification of content into complexes where it was not before, and also be withdrawn from others. But no accumulation of external counter-evidence can modify it, unless the content of the complex on which it rests is itself modified. You cannot get rid of 'Two straight lines cannot enclose a space' unless you can alter the conception of the lines or of the spatial world."[1]

1. *Implication and Linear Inference*, p. 6.

Though all implication is a priori and what is implied by a true system is true, no statement is absolutely true. All judgments are essentially corrigible. There are no facts known finally and once for all. The so-called facts are stated, and re-stated—a process of modification which never ceases until the whole of truth is attained. In the course of reasoning all statements of facts come to be modified.

Science evidently lays great stress on correction and modifiability of statements. There is a certain chance of mistake somewhere, and its method does not eliminate absolutely all chances of error. The scientist knows well that his eyes may deceive him and that the statements which are made with perfect confidence by the ordinary eye-witness have frequently to be corrected. There is nothing in the scientific method to guarantee absolute reliability to the record or statement of any one observation. The mere fact that science is glad to duplicate its observations is evidence that it does not regard any statement as absolutely incorrigible.

Let us now consider the statements of ordinary life. Can they be taken as absolutely true ? To take an example, can the statement 'This patch is blue' be taken to be true and absolutely incorrigible statement ? It may be pointed that the rough statements of the type 'this is blue' are not statements of immediately apprehended brute facts. They involve some more or less trustworthy process of inference or argument of some kind. If when looking at an object I say that is blue, my apprehension is discriminative and specific. Distinguishing it from other shades, I am apprehending that 'it is such and such an exact shade'. But no one would claim that discrimination of the exact shade was absolutely certain. Such statements would always be made provisionally and would be recognized to be corrigible in the light of subsequent considerations. These statements involve a process of 'judging' over and above thc actual perspective apprehension, and that the result of that judging, i.e., the statement, is always corrigible. They are never 'statements of apprehension' in the sense of stating just what is apprehended; rather they are statements made *on the strength of* an apprehension. Thus the making of statements of this type involves thinking of the kind which is called systematic. They are implications within a system. The truth of any of these statements depends upon its distinctive

coherence within the given system together with the whole system of reality.

Let us now turn our attention to mathematics. In geometry we start with certain first principles or axioms and make the whole system by the process of deduction. No doubt all the statements of geometry taken together form a system; but this is not enough to show that geometrical thinking is systematic in the sense of the idealistic logic Can it be shown that in geometry, as in physics, the statements of first principles are provisional, corrigible statements which need to be modified as more knowledge is acquired ? Do they not rather appear to be final, incorrigible statements of apprehensions of single facts, apprehended independently of their place in the system ?

The coherence theory sees that it is difficult to maintain that the statements of mathematics are corrigible in the sense stated. But it goes one step further and maintains that the *meanings* attached to the statements are continually modified and corrected. It points out that the statement 'Two and two make four' means something quite different to the expert mathematician and to the school boy. The meaning of the statement varies according to a man's knowledge of the context. Statements *mean different* things to different people, and to the same person at different times. Thus on the coherence theory even the statements of mathematics are corrigible in a deeper sense. Their meanings are changing and developing by constant correction.

But if a statement has no fixed meaning, it could not in itself imply anything; what was implied would depend on what was meant. Inference is then not possible, since it depends upon correct statement with fixed meaning. Thus the game is up for logic. As soon as we allow that a statement has in itself no meaning, since meaning may change without any change in the statement, then the examination of statements, which is the work of logic, must lose its value.. In allowing that a statement has no fixed meaning of its own, logic destroys its own foundation.

But why did the idealistic logic take this step? The reason is that the foundation of the coherence theory is that statements are essentially modifiable and corrigible, and as there seems to be no evidence to support this contention, the idealistic logic had no other alternative but to maintain that mathematical state-

ments are modifiable in a different sense. The analysis of mathematical reasoning does not seem to support the idealistic thesis that thinking is essentially systematic and modifiable. On the other hand, it seems to support the older view that facts may in some cases be apprehended one by one, independently of any apprehension of the connections between facts; and that the earlier apprehensions are complete and final apprehensions, totally independent of the latter, and entirely unmodifiable. The main difference between the older logic and the idealistic logic is that while the former lays stress on independent apprehension, the latter lays supreme stress on judging as the process of drawing inferences on the basis of systematic implication.

It will be good to consider Cook Wilson's views at this point. He criticises idealistic logic and defends the fundamental principles underlying the traditional position. While the idealistic logic explains all knowledge in terms of judgments, Cook Wilson must do so in terms of *apprehensions*. The idealistic theory of judgment lays great emphasis on the essential unity of all judgments, considered as forming one whole, and explains the relation between different judgments as systematic implication. Against this view, Cook Wilson lays emphasis on the *independence* of each apprehension which directly apprehends the reality, and is different from belief or opinion. It is sometimes true that we could not be having a certain apprehension unless the other apprehensions had gone immediately before. But apprehensions owe their character and nature as apprehensions not at all to the events which preceded them in mind, though the fact of their occurrence they may in part owe to them. Every apprehension is a direct and and immediate apprehension of reality, and there is nothing like mediate or indirect apprehension. There cannot be any apprehension in the absence of the thing apprehended. The so-called mediate apprehension depends for its possibility on previous apprehensions, not in the sense that those previous apprehensions by themselves force it upon the mind without the presence of the object, but rather in the sense that I cannot apprehend that a particular flower is red unless I have previously apprehened what it means to be red. The apprehension that this flower is red is quite distinct from the apprehension of what it means to be red though the apprehension depends for its occurrence on the other.

Now what are we to say about inference? Cook Wilson gives

a *descriptive* account of it. It is *not* a manifestation of *a priori* insight into the way in which knowledge *must* grow, but is a series of immediate apprehensions of reality; the conclusion represents an apprehension of reality distinct from the apprehensions expressed in the premises, though it depends for its occurrence on those apprehensions having immediately preceded it. Its occurrence is made possible by the immediate past, but its character is affected not in the least by the past. There are no necessary connections between apprehensions, but there are necessary connections between the facts apprehended. There are not necessities of apprehensions, but only *apprehensions of necessities.* What at first sight appears to be a necessary connection between apprehensions is really an apprehension of a necessary connection between facts. But what, on this view, are we to say of hypothetical reasoning, and in particular of the *reductio ad absurdum* proof? This proof, the validity of which none denies, far from stating facts, states what is palpably absurd, and its validity seems to depend upon necessary connections between statements.

Cook Wilson's account throws no light whatever on the validity of inference. Inference, it is maintained, is a series of the immediate apprehensions of reality. But what is the guarantee of the validity of these apprehensions? Can logic throw any light on the question whether these apprehensions are really apprehensions ? Can apprehension be distinguished from non-apprehension? Cook Wilson does not answer these questions. He simply takes it to be granted that there are valid immediate apprehensions which are the foundations of all knowledge.

On Cook Wilson's view, in inference apprehension of the facts stated in the conclusion depends for its *occurrence*, but not for its character, on the apprehension of the facts stated in premises. But there cannot be an occurrence of *mere* apprehension *devoid of* all character. Occurrence is always an occurrence of a particular kind of apprehension. The apprehensions of the facts stated in the premises determine not only the occurrence of the facts stated in the conclusion but also its *character*. There is the *necessary relation of implication which holds* between the premises and the conclusion, and this relation is not a temporal sequence but a logical relation which determines what the conclusion must be.

If the character of the apprehension of the facts stated in the premises is independent of that stated in the con-

clusion, how can we explain the necessary (logical) connection between the two? None can deny that there is a necessary connection between them, the conclusion necessarily follows from the premises. This necessary connection cannot be the apprehension of a necessary connection between facts for implication holds independently of any fact. A valid hypothetical reasoning is of the form 'if *p*, then *q*' Here *p* and *q* are propositions or statements but not statements of facts. *p* and *q* would become statements of facts only if they are true. But in a syllogism the statement of propositions as premises and conclusion is independent of their truth-values. Therefore the necessary relation (which is callled implication) between them is independent of any fact and cannot be the apprehension of any connection between facts.

The metaphysical theory of implication is better than the traditional theory of linear implication, for the former explains more successfully the necessity involved in implication. According to it, the necessity is the necessity of the Absolute system. If we accept the premises, we must accept the conclusion, otherwise the whole system will fall down. But the metaphysical theory swallows the distinction between different systems and between the ideal and the factual aspects of reality in which common sense has much faith. This theory does not conform to the notion of implication as it is ordinarily understood and, therefore, even if the Absolute System exits, we cannot know the logical relation between its different units. The logic of the Absolute is not given to us.

5

Material and Formal Implication

The conception of Material Implication[1] occurs in Principia Mathematica. This conception is quite consistent with the mathematical system in which it occurs. But the authors of Principia Mathematica were mainly interested in constructing a deductive system. Hence they have sacrificed common usage of the term to the desiderata of system building, such as, consistency and independence of primitive propositions, minimum of fundamental notions, and deductive power of primitive propositions. Their definition of implication entails what have come to be known as the 'Paradoxes of Implication'. It fails to define that precise relation which exists between the premises of a valid deduction and its conclusion in ordinary reasoning.

The definition of implication as given in Principia Mathematica is :

$$(p \supset q) = (\sim p \vee q) \text{ Df.}$$

i.e., '*p* implies *q*' is defined to mean 'either *p* is false or *q* is true'. Here implication is defined in terms of disjunction. A disjunctive proposition may have at least three different meanings. When we say 'either *p* is true or *q* is true' it may mean that both *p* and *q* cannot be true. But such a meaning is ruled out because disjunctions in the logical system of Principia Mathematica do not signify mutual exclusion. If *p* be true, it is not implied

1. Russell uses 'implication' without the prefix 'material' in *Principia Mathematica* which in the *Principles of Mathematics* he calls 'material implication'.

that q is false. Both p and q may be true. Two meanings of disjunction still remain to be considered.

To understand these two meanings let us compare the following disjunctions :

(1) Either 'Cows have no wings' or 'Delhi is in India'.
(2) Fither Sitā does not love me' or 'I am beloved'.

The first disjunction is such that *at least one* of its propositions is *as a matter of fact* true; the second disjunction is such that at least one of the disjoined propositions is 'necessarily' true. If one of the lemmas in the second disjunction is false, we are in consistency bound to suppose the other true. But if the proposition 'Delhi is in India' were false, it would not bind us to suppose that the proposition 'Cows have no wings' is true.

The most significant distinction between the two disjunctions is that the first states a material truth, while the second states a formal truth. The truth of the first depends upon the facts, but the truth of the second is independent of the facts in question. The sccond disjunction is such that its truth is independent of the truth of either member considered separately. Its truth can be known while it is still problematic which of its lemmas is the true one. The truth of 'Either Delhi is in India or cows have no wings' lacks this purely formal character and independence of facts.

We may call disjunctions like the first, whose truth cannot be known apart from the facts, extensional disjunctions; those of the second type which state formal truth and whose truth can be known apart from the facts in question, we may call intensional disjunctions. We may further distinguish these two types of disjunctions by considering their negatives. The negative of an extensioal disjunction is the denial of both its members; the negation of an intensionanl disjunction is the negation of the *logical relation* between its propositions. Though there is a logical connection between the truth-values of the two propositions in an intensional disjunction, the truth or falsity of either member does not affect the *necessary logical relation* between them. Whether Sitā loves me or does not love me, the logical relation asserted in the proposition 'Either Sitā does not love me or I am beloved', is not affacted at all.

In Principia Mathematica the meaning of disjunction is confined to the extensional. In it the negative of a disjunction is the negation of both its members, [$\sim(p \vee q) = \sim p \sim q$], and the negative of a product is the disjunction of the negatives of its factors, [$\sim(pq)=(\sim p \vee \sim q)$]. Every intensional disjunction is also extensional but every extensional disjunction is not intensional. In other words, intensional disjunction of p and q implies their extensional disjunction also but the reverse does not hold. It is the extensional meaning of disjunction in the definition of material implication that has led to the 'Paradoxes of Implication'.

Since an extensional disjunctive proposition is true when one of the disjuncts is true, it follows that

(a) $p \supset q$ is true whenever p is false,

i.e., "a false proposition implies every proposition," and also that

(b) $p \supset q$ is true whenever q is true,

i.e., a true proposition is implied by every proposition.

Other characteristic properties of material implication are the following:

(1) $(p \supset q) \vee (p \supset \sim q)$,

i.e., given any two propositions p and q, either p implies q or p implies not-q.

(2) $(p \supset q) \vee (q \supset p)$,

i.e., of any two propositions, either the first implies the second or the second implies the first. In other words, no pair of propositions can be independent.

(3) $(p \supset q) \vee (\sim p \supset q)$,

i.e., if p and q are any propositions, either q is implied by p or it is implied by not-p.

(4) $pq \supset [(p \supset q) . (q \supset p)]$,

i.e., of any two true propositions, p, q, each implies the other.

(5) $(\sim p \sim q) \supset [(p \supset q) . (q \supset p)]$,

i.e., of any two false propositions, each implies the other.

(6) $\sim p . q \supset (p \supset q)$,

i.e., of any two propositions, one of which is false and the other true, the false one implies the true one.

The fallacy of these theorems consists in asserting the relation of material implication between totally independent propositions. Take any true proposition, e.g., 'Kālidāsa wrote Śākuntalam', it is (materially) implied by any proposition such as 'Delhi is in India', or 'Socrates is a triangle' because the extensional disjunctions—'Either Delhi is in India or Kālidāsa wrote Śākuntalam' and 'Either Socrates is a triangle or Kālidāsa wrote Śākuntalam'—hold good. But the relation between the members of these disjunctions is not the same relation which holds between two propositions when one can be deduced from or is entailed by the other. It would not be commonly maintained that 'Kālidāsa wrote Śākuntalam' can be deduced from 'Delhi is in India' or from 'Socrates is a triangle'. There can be no doubt that the relation of material implication is quite different from entailment or ordinary implication. But Russell regards the relation of material implication as the sole relation in virtue of which deduction is possible. Thus he says, "The relation in virtue of which it is possible for us to infer validity is what I call material implication."[1] To support this view he argues, "It would certainly not be commonly maintained that '2+2=4' can be deduced from 'Socrates is a man', or that both are implied by 'Socrates is a triangle'. But the reluctance to admit such implications is chiefly due, I think, to preoccupation with formal implication, which is a much familiar notion, and is really before the mind, as a rule, even where material implication is what is explicitly mentioned."[2] Certainly the extensional disjunction 'Either Socrates is a triangle or 2+2=4' holds good, since in fact the first member of the disjunction is false and the second is true. But is is clear that '2+2=4' cannot be deduced from 'Socrates is a triangle'.

The theorems of material implication are not correcly applicable to this world. They are not useful as modes of inference and proof in this world. They are only propositions about the nature of some other world to which the system of material implication would apply. The main characteristics of such a world would be the following :

(1) The all-possible must be real. The disjunction of any two propositions, one of which happens to be true, e.g., 'Either cows

1. *Principles of Mathematics*, p. 33.
2. *Ibid.*, p. 34

have wings or Delhi is in India' must have the force of a dilemma whose alternatives exhaust the possibilities.

(2) The true mast be necessarily true, true a priori. Since a true proposition would be implied by any proposition, it would be implied even by its contradictory.

(3) The contingent could not exist, since all facts would be necessarily true.

(4) The false would be impossible and absurd. Every false proposition would (materially) imply its own contradiction and would thus be proved absurd.

(5) The contrary-to-fact supposition, being false, would be impossible and absurd.

(6) The world would be marked by the omnipresence of the relation of implication because a true proposition would be implied by every other proposition.

Practically the relation of material implication is useless. To assert that 'p materially implies q' we must first know the truth-values of p and q. In order to use this relation we will need to know the truth or falsity of the propositions concerned. If we are asked what the consequences (in the sense of material implication) of p are, we cannot say q without knowing the truth or falsity of both p and q. Even if we know the truth-values of 'p and q' and assert that 'p (materially) implies q', q cannot be said to be the consequence (in the ordinary sense) of p. On the basis of our knowledge that p is true, we cannot say that q is necessarily true. Before asserting the relation of material implication we must know the truth-value of q independently. Take an example, 'Either the Himālayas are in India or Keats died'. 'The Himālayas are in India' materially implies 'Keats died'. But before asserting this relation, we already know that both the propositions 'The Himālayas are in India' and 'Keats died' are true. On the basis of our knowledge that the Himālayas are in India', we can never assert that Keats died.

But to assert the ordinary relation of implication between propositions, it is not necessary at all to know their truth-values. What is required to assert such a relation is the necessary logical connection between the propositions. Take the following example:

Rāma is older than Mohan and Mohan is older than Shyāma, ...(1)
Therefore, Rāma is older than Shyāma...... (2)

The relation of implication holds between the propositions (1) and (2) irrespective of their truth-values. To assert the relation of implication we are not at all concerned with their truth or falsity but only with the logical connection between their meanings. But if after asserting this relation we know that (1) is true, we can immediately infer that (2) is true. Ordinary implication implies material implication. In other words, every case of ordinary implication is a case of material implication (provided the corresponding factual situation occurs). We can say *p* materially implies *q* in the following cases:

p is true and *q* is true
p is false and *q* is true
p is false and *q* is false

p does not materially imply *q* when *p* is true and *q* is false. Before asserting this relation we must know the truth-values of both propositions. We can calculate the degree of chance of any proposition materially implying any other proposition. "Let an equal number of true and false statements chosen at random, and regardless of their subject matter be written on slips of paper and put in a hat. Let two of these then be drawn at random. The chance that the first drawn will mater'ally imply the second is $\frac{3}{4}$. The chance that the second will materially imply the first is $\frac{3}{4}$. The chance that each will materially imply the other $\frac{1}{2}$. And the chance that neither will materially imply the other is o"[1]

To assert the relation of ordinary implication between any two propositions we need not know their truth-values but only the necessary logical connection between them. But if the relation of ordinary implication is asserted between two propositions, then

If *p* is true, *q* is true
If *p* is false, *q* may be true or false
If *q* is true, *p* may be true or false
If *q* is false, *p* is false.

It is important to notlce that the relation of material implication does not furnish us with sufficient apparatus for the actual

1. Lewis & Langford, *Symbolic Logic (1932)*, P. 145.

drawing of a categorical inference in any given case of ordinary reasoning. In order to infer q from p we must know that p is true, and that p implies q, and must *not know* q is true. But material logic[1] cannot furnish us with any such a priori knowledge of the relation of implication between propositions without knowing their truth-values. If we already know that q is true, inferencc is useless and unnecessary. It is only intensional logic which provides and explains correctly the formal relation of implication needed for inferring q from p without appeal to facts.

The uselessness of materical logic can be proved by an example from the field of induction. Suppose we have an hypothesis H_1 to explain a certain fact. Since an hypothesis can be verified only by testing its implications, we must see what its consequences are. Material logic cannot furnish us with any implications of a proposition because, according to it, the material implications of a proposition depend upon its truth-values. But we do not know whether H_1 is true or false. Hence no implications can be drawn. To draw its consequences we have to suppose either that H_1 is false or that H_1 is true. If H_1 is false, it materially implies every (even false) proposition, and is thus, according to the common rules of inference and implication, disproved. If we suppose that H_1 is true, it implies every true but no false proposition, and hence is verified. But in both the cases our reasoning has been circular. Hypotheses are not given to us as true or false. They are mere provisional suppositions to explain certain facts. If the implications of an hypothesis are proved to be true, it is verified. If it has false consequences, it is not verified. And we make another hypothesis H_2. The usefulness and validity of the method of hypothesis lies in the fact that the consequence of an hypothesis may be drawn independently of truth-value. The drawing of consequences presupposes an intensional logic such that an examination of the intrinsic content of a proposition, apart from consideration of its truth-value, is sufficient to the deduction of its consequences. Inference and implication depend upon meaning, logical import, intension.

In spite of the uselessness of material implication for practical purposes, we maintain that the concept is consistently used in the logical system of *Principia Mathematica*. There are no paradoxi-

1. We are using here thc phrase 'material logic' for 'extensional logic'.

cal propositions that assert that a false proposition implies every proposition, or that a true proposition is implied by any proposition. The logical system is purely abstract. The paradoxes arise if we interpret the symbol '⊃' as 'implies'. That the authors have interpreted it as such is clear from the following note to the proof of *2.16 in *Principia Mathematica* (p. 103, Vol. I, 2nd ed.):

'The proposition to be proved will be called "Prop", and when a proof ends, like that of *2.16, by an implication between asserted propositions, of which the consequent is the proposition to be proved, we shall write "⊢ . etc.⊃ ⊢ .Prop". Thus '⊃ ⊢ .Prop" ends a proof, and more or less corresponds to "Q.E.D."

From this note it is clear that the notation '⊃ ⊢ .proposition' is to mean that the proposition to be proved is a consequence of propositions already asserted. That is to say, the horse-shoe '⊃' symbolises the relation 'is a consequence of'. There are other examples of reference to implication which is not merely 'material'. Note the following quotation from *Principia Mathematica* (Vol. I, p. 90) :

"Now in order that one proposition may be inferred from another, it is necessary that the two should have that relation which makes the one a consequence of the other. When a proposition *q* is a consequeuce of a proposition *p*, we say that *p implies q*."

It is interesting to note that in explaining 'Descriptions' in his *Introduction to Mathematical Philosophy*, Russell uses a form of implication which is not material when he says of the three propositions, "at least one person wrote Waverly", "at most one person wrote Waverly", "whoever wrote Waverly was Scotch", that the three together (but no two of them) imply that the author of Waverly was Scotch. Either two of the three, or any of them alone, materially implies the conclusion because, according to material implication, true propositions imply each other. But Russell says that 'the three together (but no two of them)' imply the conclusion. Here the notion of implication is similar to entailment or ordinary implication.

It may be pointed out that the authors' interpretation of '⊃' as 'implies' is unofficial. But the horse-shoe (⊃) is used in a consistent manner and there is nothing wrong if we regard '$p \supset q$' as a mere abbreviation for '$\sim p \vee q$'. In that case the relation between

the primitive propositions and the theorems would be simple such that either the primitive propositions are false or the theorems are true, and the latter need not be the consequences of the former. But this is incompatible with the fundamental aim of the authors that the theorems follow from the primitive propositions.

The logical system of *Principia Mathematica* may be compared to the non-Euclidean system of geometry. Both the Euclidean and the non-Euclidean systems of geometry are self-consistent. But when it becomes necessery to choose one, we prefer the Euclidean system for the following reasons; (1) Both are self-consistent systems and their theorems, as pure mathematics, equally true, but they are true of different kinds of space. Since our space is actually Euclidean, the non-Euclidean system is false as applied geometry. (2) The Euclidean system accords with our practical and useful ways of dealing with space-filling objects. It is thus practically acceptable to us and is pragmatically the true one.

Like the non-Euclidean geometry, the logical system of *Principia Mathematica* is not applicable to our ordinary modes of inference and proof.

Formal Implication

On the basis of material implication, Russell has developed the notion of *formal implication*. By formal implication he means *general material implication*. For Example" 'X is a man' materially implies 'X is mortal' for all values of X" states a formal implication. In other words, " 'X has the property of being man' materially implies 'X has the property of being mortal', whatever X may be". It is to be noted that "'X is a man' materially implies 'X is mortal'", is a single propositional function and is not a relation of two propositional functions. We must assign the same value to X on both sides of the implication. We cannot first vary X in 'X is a man' and then independently vary it in 'X is mortal', for this would lead to the proposition that 'everything is a man' materially implies 'everything is mortal', which was not what was meant. By assigning the same value on both sides of the implication we would obtain cases of material implication such as "Rāma is a man materially implies Rāma is a mortal ", "Krishṇa is a man materially implies Krishṇa is mortal", etc. Thus formal implication asserts a class of material implications, not a single implication. Here we do not have

one implication containing a variable, but rather a variable implication.

The proposition "X is a man materially implies X is a mortal, for all values of X" is equivalent to "all men are mortal", "every man is mortal" and "any man is mortal". These are general propositions in which a certain complex notion 'man' is asserted to be a constituent of another complex notion 'mortal'. If anything is man, it is mortal. But Russell's interpretation of general propositions may be said to be extensional because, according to him, it asserts a class of material implications; it asserts that in every case of a certain 'set of cases' material implication holds. If such an interpretation is accepted, we have to know the truth-value of each case before asserting formal implication. Since it is not possible to know the truth-value of every case of class 'man', formal implication concerning such a class (whose members are unlimited) cannot be asserted. It can be asserted only regarding a *small* class after knowing the truth-value of *each* case. Like material implication, such an extensional formal implication is useless for inference. But Russell wrongly thinks that it is useful for drawing inference. He says, "one of our indemonstrable principles was, it will be remembered, that if the hypothesis in an implication is true, it may be dropped, and the consequent asserted................ The principle is employed whenever a proposition is said to be *proved*; for what happens is, in all such cases, that the proposition is shown to be implied by some true propositions. Another form in which the principle is constantly employed is the substitution of a constant, satisfying the hypothesis, in the consequent of a formal implication. If ϕx implies ψx for all values of x, and if 'a' is a constant satisfying ϕx, we can assert ψa dropping the true hypothesis ϕx."[1] But formal implication, being a class of material implications, can be asserted only after knowing the truth-value of each case. If we already know that ψa is true, then to assert it again on the basis of formal implication is circular reasoning which is useless.

Mathematical propositions, according to Russell, assert formal implication. "We assert always in mathematics that if a certain assertion p is true of any entity x, or any set of entities x, y, z,......, then some other assertion q is true of those entities; but we do not assert either p or q separately of our entities. We assert a relation

1. *Principles of Mathematics*, p. 34.

between the assertions *p* and *q*, which I shall call formal implication."[1] An example from geometry will make Russell's contention clear. The fifth proposition of Euclid states that if x is an isosceles triangle, x has the angles at its base equal. But has Russell examined all cases of isosceles triangles and known that the angles at the base of all isosceles triangles are equal ? Since formal implication asserts a class of material implications, the propositional function 'If x is an isosceles triangle, x has the angles at the base equal, for all values of x', cannot be asserted without knowing the truth-value of each case. It is obvious that mathematical propositions are not asserted by knowing the truth-value of each case. They are asserted a priori on the basis of the premises from which they follow in virtue of the concept or meaning. The relation between the premises and the conclusion is intensional.

1. *Principles of Mathamatics*, p. 5.

6

Strict Implication

C. I. Lewis has developed a system of implication which he calls strict implication. He holds that the notion of material implication is an extensional one and is not applicable to our ordinary modes of inference and proof, while the notion of strict implication is an intensional one and is applicable throughout our inference. The notion of material implication is extensional because it is defined by its authors in terms of truth-values of the propositions. *p* materially implies *q* is defined as 'either *p* is false or *q* is true', $(p \supset q) . = . (\sim p \vee q)$ Df. We cannot know whether *p* materially implies *q* or not without knowing the truth or falsity of *p*, *q* separately. But Lewis claims that this is not the case with strict implication. We can know whether *p* strictly implies *q* or not without knowing their truth-values.

Strict implication is defined as follows:

$p \prec q = \sim \Diamond(p \sim q)$[1]

('$\Diamond$' is the symbol used by Lewis for possibility or self-consistency).

This definition is to be read as "*p* stricty implies *q*" means "it is impossible for *p* to be true and *q* false" or "the statement '*p* is true and *q* is false' is not self-consistent". When *q* is deducible from *p*, to say '*p* is true and *q* false' is to assert, implicitly, a self-contradiction.

Lewis[2] holds that while the extensional interpretation of the disjunction either '*p* is false or *q* is true' produces material implication, the intensional interpretation of the same produces

1. Lewis and Langford, *Symbolic Logic*, p.124.
2. See his article 'Implication and the Aljebra of Logic, *Mind*, 1912.

strict implication. An extensional disjunction is satisfied by the *material truth* of one of its members. It is not necessary that there should be any relation between their meanings. In an intensional disjunction, if either member were false, the other would *necessarily* be true because there is a necessary relation between their meanings. In every case of extensional disjunction intensional disjunction does not hold, while extensional disjunction does hold in every case of intensional disjunction. Take, for example, the proposition "'Either this cloth is not red' or 'Nehru wrote *The Discovery of India*'". It asserts extensional disjunction because it is true that Nehru wrote *The Discovery of India.* But it is not an intensional disjunction because there is nothing in its meaning, apart from the facts, on the basis of which we can know its truth. There is no necessary relation between the meanings of its members. Let us consider another proposition, "'Either Sītā does not love me' or 'I am beloved'". This proposition asserts an intensional disjunction because there is the intrinsic connection between the meanings of its members and we know its *formal truth* apart form the facts. But it also asserts an extensional disjunction as at least one of the disjoined propositions is necessarily, as a fact, true. The proposition 'Sītā loves me' strictly as well as materially implies 'I am beloved'. But the proposition 'This cloth is red' does materially, but not strictly, imply the proposition 'Nehru wrote *The Discovery of India*'. Lewis maintains that it is in virtue of the relation of strict implication only that deduction is possible. From the proposition 'Sītā loves me' we can deduce 'I am beloved' because there is the relation of strict implication between them. But from the proposition 'This cloth is red' we cannot deduce 'Nehru wrote *The Discovery of India*' because the relation of strict implication does not hold between them. Every case of strict implication is a case of material implication. This can be symbolically expressed as follows:

$$p \prec q . \prec . p \supset q$$

$$p \prec q . \prec . \sim(p \sim q)$$

That is, if p strictly implies q, then it is not the case that p is true and q false. But its converse

$$\sim(p \sim q) . \prec . p \prec q$$

does not hold. "Strict implication is a *narrower* relation than material implication; the assertion of a strict implication is a stronger

statement than the assertion of the corresponding material implication."[1]

In spite of important differences between the two relations, there are some theorems which hold of both relations. For comparison such theorems may be exhibited together, keeping in mind that the symbol 'v' is used for *extensional disjunction* with regard to material implication and for *intensional disjunction* with regard to strict implication.

Material Implication	Strict Implication
$p \supset q. = .\sim p \vee q$	$p ⥽ q. = .\sim p \vee q$
$p \vee p. \supset .p$	$p \vee p. ⥽ .p$
$q. \supset .p \vee q$	$q. ⥽ .p \vee q$
$p \vee q. \supset .q \vee p$	$p \vee q. ⥽ .q \vee p$
$p \vee (q \vee r) .\supset .q \vee (p \vee r)$	$p \vee (q \vee r) .⥽ .q \vee (p \vee r)$
$p \supset q. q \supset r : \supset .p \supset r$	$p ⥽ q. q ⥽ r : ⥽ .p ⥽ r$[2]
$p \supset q. = .\sim q \supset \sim p$	$p ⥽ q. = .\sim q ⥽ \sim p$

Material implication, like strict implication, is subject to the law that its terms may be interchanged by negating both.

The following theorems[3] show the connection between material and strict implication:

$$pq. ⥽ .r: = :p. ⥽ .q \supset r: = :q. ⥽ .p \supset r$$

$$q \supset r. ⥽ :p \vee q. \supset .p \vee r$$

$$p .p \supset q: ⥽ .q$$

$$p ⥽ q. ⥽ .p \supset q$$

$$p \supset q. q \supset r: ⥽ .p \supset r$$

$$p \supset q. ⥽ .\sim q \supset \sim p$$

Let us now see the differences between the two. There are some theorems which hold of material implication but not of strict implication. The definition of material implication may be written in the following form:

$$p \supset q = \sim(p \sim q)$$

1. Lewis and Langford, *Symbolic Logic*, p. 137.
2. If p implies q, and q implies r, then p implies r. This is sometimes referred to as the Principle of Syllogism.
3. See *Ibid.*, pp. 138-41.

This definition has the following force:

Whenever p materially implies q, it is not the case that p is true and q false. . . . (1)

Whenever it is not the case that p is true and q false, p materially implies q. . . . (2)

The analogue of (1) for strict implication holds, that is, whenever p strictly implies q, it is not the case that p is true and q false.

But the analogue of (2), that is, whenever it is not the case that p is true and q false, p strictly implies q, does not hold:

$$\sim(p \sim q) \neq p \prec q$$

These elements (1) and (2), contained in the definition of material implication, are the sources of all the related paradoxes.

$$p. \supset .q \supset p$$

That is, if p is true, then any proposition q materially implies p. But its analogue for strict implication:

$$p. \prec .q \prec p$$

cannot be proved.

$$\sim p. \supset .p \supset q$$

That is, if p is false, then p materially implies any proposition q.

But, as before, its analogue for strict implication:

$$\sim p. \prec .p \prec q$$

cannot be proved.

$$\sim(p \supset q). \supset .p \supset \sim q$$

That is, if p does not materially imply q, then p materially implies $\sim q$. Its analogue:

$$\sim(p \prec q). \prec .p \prec \sim q$$

cannot be proved, and is false.

$$\sim(p \supset q). \supset .q \supset p$$

That is, if one of a pair of propositions does not materially imply the other, then the other implies the one. The analogue for strict implication:

$$\sim(p \prec q). \prec .q \prec p$$

does not hold.

$$\sim(p \supset q) . \supset . \sim p \supset q$$

That is, if p does not materially imply q, then the denial of p implies q. But the analogue, $\sim(p \prec q) . \prec . \sim p \prec q$, does not hold.

$$\sim(p \supset q) . \supset . q \supset \sim p$$

That is, if p does not materially imply q, then q implies that p is false. The analogue, $\sim(p \prec q) . \prec . q \prec \sim p$, does not hold.

$$p \supset q . \vee . p \supset \sim q$$

That is, at least one of the two, 'p materially implies q' and 'p materially implies that q is false', is always true. The analogue

$$p \prec q . \vee . p \prec \sim q$$

cannot be proved.

$$pq . \supset . r : \supset : p . \supset . q \supset r$$

That is, if two premises materially imply a certain conclusion, and one of these premises is true, then the conclusion is materially implied by the other premise alone. But its analogue

$$pq . \prec . r : \prec : p . \prec . q \prec r$$

does not hold. If two premises strictly imply a certain conclusion, and one of these premises is true, it does not generally follow that the conclusion can be deduced from the other premise alone.

All the functions of p and q which figure in the system of material Implication are *truth-functions* of p and q. The truth or falsity of any one of these functions is determined by the truth or falsity of p and q. But the relation of strict implication is not such a truth-function. $p \prec q$ cannot be determined merely by knowing whether p, and q are true or false.

While defining strict implication, Lewis has introduced the notion of *impossibility* or *self-contradiction*. His definition of strict implication, as stated above, is the following.

$$p \prec q . = . \sim \Diamond (p \sim q)$$ [1]

1. Lewis and Langford, *Symbolic Logic*, p. 124.

That is 'p strictly implies q' means that 'it is impossible for p to be true and q false'. The extensional disjunction, $\sim p \vee q$, that is, 'either p is false or q is true' which is equivalent to 'it is not the case that p is true and q false', seems to be converted, by Lewis, into intensional disjunction by introducing the notion of 'impossibility'. Although this notion is not explicitly defined, its meaning can be shown by the notions that are defined in terms of it.

$$\Diamond p. = .p \circ p$$

i. e., 'p is possible' means the same as 'p is self-consistent'.

17.1 $\quad \Diamond pq. \prec .p \circ q$[1]

i. e, any two propositions, both of which are true, must be consistent. It should be noted that pq is not the definition of $p \circ q$. The implication asserted here is not reversible; two propositions may be consistent though one is false and the other true, or though both are false.

17.01 $\quad p \circ q. = .\sim(p \prec \sim q)$[2]

i. e., 'p is consistent with q' means that 'it is false that p implies the falsity of q', which has exactly the same force as 'p does not imply that q is false'. Lewis observes, "The properties which result from this definition will serve as a check upon the accord between the usual meanings of 'implies' and of 'consistency' and the functions $p \prec q$ and $p \circ q$."[3] The following proposition shows the relation between strict implication and consistency:

17.12 $\quad p \prec q. = .\sim(p \circ \sim q)$[4]

That is, 'p strictly implies q' means that 'p is inconsistent with the denial of q'.

Compare it with the definition of material implication.

$$p \supset q. = .\sim(p \sim q)$$

'p materially implies q' means that 'it is not the case that p is true and q false'. "Let p = 'Roses are green' and q = 'Sugar is

1. Lewis and Langford, *Symbolic Logic*, p. 154.
2. *Ibid.*, p. 153.
3. *Ibid.*, p. 153.
4. *Ibid.*, p. 154.

sweet'. Since p is false (or since q is true), $\sim(p \sim q)$ holds for this example; and hence $p \supset q$ holds: 'Roses are green' materially implies 'Sugar is sweet'. But, so far as one can see, 'Roses are green' and 'Sugar is not sweet' are not inconsistent with one another: $\sim(p \circ \sim q)$ does not hold, and hence $p \prec q$ does not hold."[1]

From this it is clear that Lewis is trying to distinguish between material and strict implication by maintaining that in any case of material implication $\sim(p \sim q)$ holds as a matter of fact, but considering the same case for strict implication $\sim(p \circ \sim q)$ may not hold, that is, p may not be inconsistent with $\sim q$. But what is the exact meaning assigned to the word 'impossibility' or 'inconsistency' by him? From the above it is clear that for him the notion of 'possibility' is equivalent to 'sef-consistency', and 'impossibility' is equivalent to 'inconsistency' or 'self-contradiction', and his theorem 17.01 (*Symbolic logic*) defines consistency in terms of truth-values and implication. So it seems that he could not give a definition of consistency independently of truth-values.

He distinguishes between the relative and the absolute meanings of the words 'possible', 'impossible' and 'necessary'. Relative meaning has reference to the relation which the proposition or the thing considered has to some state of affairs, such as given data, or to our knowledge as a whole. In this sense, 'possible' means 'consistent with the data' or 'consistent with everything known'; 'impossible' means 'not consistent with the data, or with what is known'; and 'necessary' means implied by what is given or known'. Lewis claims to symbolize *absolute* meanings. Absolute meaning is concerned only with the relation which the fact or proposition has to itself or to its negative—what can be analysed out of the proposition by sheer logic. But truth or falsity of a proposition cannot be analysed out of proposition by sheer logic. Then how he can justify his statement: "Any two propositions, both of which are true, must be consistent. This implication is not reversible: two propositions may be consistent though one is false and the other true, or though both are false." In this statement he makes no reference to the meanings of the propositions concerned. What can be analysed out of the proposition and can be known without reference to fact is the meaning of the proposition and not its truth-value. If the meaning of an analytic proposition is not self-contradictory, it would be said to

1. Lewis and Langford, *Symbolic Logic*, p. 154.

be self-consistent. Truth or falsity is the property of only synthetic or empirical propositions and can be determined only with reference to facts. Proper meaning of the word 'impossible' or 'inconsistent' for a truly intensional logic would be 'self-contradictory *in meaning*' or 'logically inconceivable'.

To examine further Lewis's meaning of 'consistency', 'impossibility' and 'necessity', let us take the following theorems :

18.4 $\quad p \prec \Diamond p$

that is, what is true is logically possible or conceivable.

18.41 $\quad \sim\Diamond p \prec \sim p$

i.e., what is impossible is false.

18.42 $\quad \sim\Diamond\sim p \prec p$

i.e., what is necessary is true.

19.1 $\quad \sim(p \circ p) \prec \sim(p \circ q)$

i.e., if p is not consistent with itself, then p is not consistent with any proposition.

Theorem 19.1 is referred to as the 'Consistency Postulate'. From this it follows that if p is an inconsistent or impossible proposition, it will be inconsistent with any proposition, say $\sim q$; therefore, according to theorem 17.12,[3] p will strictly imply q. In other words, an impossible proposition strictly implies any proposition. The paradox similar to it is found in the system of material implication: 'A false proposition implies any proposition'. It should be noted that in the system of strict implication what is impossible is false.

Another significant paradox in the system of strict implication is: A necessary proposition is implied (strictly) by all propositions. If q is a necessary proposition, then $\sim q$ will be impossible and therefore inconsistent with any proposition p; consequently, q will be strictly implied by p. The corresponding paradox in the system of material implication is : A true proposition is implied by any proposition. In the system of strict implication what is necessary is true.

1. Lewis and Langford, *Symbolic Logic*, p. 154.
2. *Ibid.*, pp. 163 and 167.
3. *Ibid.*, p. 154.

We have already seen that material implication is not what is ordinarily understood by implication. It can hold between two propositions, when it is not possible to deduce the one from the other. Lewis is quite concerned to show that the relation of strict implication holds between the two propositions p and q, when and only when it is possible validly to deduce q from p. Therefore, he tries to justify the paradoxes of strict implication. He says:

"From any proposition of the form $p \sim p$, any proposition whatever, q, may be deduced as follows :

Assume $p \sim p$ (1)

(1) $\prec$ p (2)

If p is true and p is false, then p is true.

(1) $\prec$ $\sim p$ (3)

If p is true and p is false, then p is false.

(2) . $\prec$. $p \vee q$ (4)

If, by (2), p is true, then at least one of the two, p and q, is true. (3). (4): $\prec$. q

If, by (3), p is false; and, by (4), at least one of the two, p and q, is true; then q must be true."[1]

From this proof it is clear that Lewis is interpreting 'impossibility' extensionally. An impossible proposition, $\sim \Diamond p$, is interpreted as the conjunctive proposition '$p \sim p$' i.e., 'p is true and p is false'. Though theorem 18.12, $\sim \Diamond p. = .\sim(p \circ p). = .p \prec \sim p$, that is, '$p$ is impossible' means 'p is not self-consistent' or 'from p, its own negation can be deduced', simply asserts that the falsity of an impossible proposition can be deduced from its truth, what an impossible proposition is apart from the fact that it is true as well as false, we do not know. Hence in the absence of any further interpretation, $\sim \Diamond p$ must be interpreted as 'p is true and p is false'. Theorem 18.41 also justifies this interpretation. Though Lewis seems to be *trying* to give an intensional definition when he says that $\sim \Diamond p$ means "p is logically inconceivable", or when he says that absolute meanings 'concern only the relation which the fact or proposition has to itself or to its negative—what can be analysed out of the proposition by sheer logic,'[1] while defining, interpreting

1. Lewis and Langford, *Symbolic Logic*, p.2.50.
2. *Ibid.*, p. 161.

and applying the notion of 'impossibility' to different theorems he runs back into extensional logic. Hence, the paradoxes arise. 'What can be analysed out of the proposition by sheer logic', is not the truth or falsity of the proposition. We cannot know the truth or falsity of a proposition by merely analysing it. Truth or falsity is a property or ratio cognoscendi, and not an essence or ratio essendi of a proposition. By analysing a proposition, we can know the relation between its different elements and ascertain whether they form a consistent set or coherent system. But nowhere in the process of analysis do we come across its truth-value which can be ascertained only with reference to facts which it intends to express. If by analysis we find that its different elements are not consistent, we can simply say that it is an *impossible* or *self-contradictory* proposition. But we cannot say that it is true, or false, or both true and false. An example will make this clear. The proposition, the three angles of a triangle are equal to 115°, is impossible or self-contradictory, because the two elements, 'being a triangle' and 'its three angles being equal to 115°', are inconsistent. In common language we often say that it is a *false* proposition. But the use of the word 'false' here is confusing and misleading. There is a difference between its falsity and the falsity of the proposition 'Cows have wings'. While falsity of the latter can be known only with reference to facts, the falsity of the former can be known by mere logical analysis. Therefore, the former should be referred to as a self-contradictory or impossible proposition.

Let us further consider Lewis's definition of strict implication in which the notion of '*impossibility*' is used :

$$p \prec q. = .\sim \Diamond (p \sim q)$$

That is, '*p* strictly implies *q*' means 'it is impossible that *p* is true and *q* is false'. This definition may have two meanings: (1) It is impossible that *p* is true and *q* false in virtue of the logical connection between their *meanings* or essences. In this sense the connection between the truth-values of *p* and *q* is a consequence of the connection between their meanings. Meaning or essence of a proposition is prior to and independent of its truth-value. (2) It is impossible that *p* is true and *q* false in virtue of their *truth-values* which are independent of one another. Lewis, it appears, has used the notion in the second sense because he justifies that an impossible proposition strictly implies any and every proposition, irrespec-

tive of any connection between their meanings. For him, if p is an impossible proposition *by itself*, then $p \prec q$ holds. In his system an impossible proposition, '2+2 = 5', implies 'grass is green'. He interprets an impossible proposition p as 'it is impossible that p is true'. What is impossible is false, $\sim\Diamond p \prec \sim p$, (theorem 18.41)[1]. It is here that an intensional notion of 'impossibility' is confused with the extensional notion of 'truth' or 'falsity'. We have already explained that where a proposition is impossible or self-contradictory, it cannot be referred to as true or false.

Since Lewis interprets intensional notions: 'impossibility', 'necessity', 'consistency' etc., extensionally, there is a striking parallel between the paradoxes of the system of strict implication and those of material implication. The following is a list of some of the parallel paradoxes (capital letters in the brackets stand for a 'paradox' in the system of material implication and small letters for one in the system of strict implication):

(A) A false proposition implies any and every proposition.
(a) An impossible proposition implies any and every proposition.
(B) A true proposition is implied by any and every proposition
(b) A necessary proposition is implied by any and every proposition.
(C) A false proposition is inconsistent with all propositions.
(c) An impossible proposition is inconsistent with all propositions.
(D) No pair of propositions can be independent.
(d) No pair of propositions, of which at least one is necessary or impossible, can be independent.

We have examined Lewis's notion of 'impossibility'. Let us now examine another significant notion, that of 'necessity' which occurs in the paradox, "a necessary proposition is implied by every proposition." Theorem 8.42 states,

$$\sim\Diamond\sim p \prec p$$

that is, what is necessary is true. Here also we find an extensional notion. What a necessary proposition is apart from its being true, we do not know in the system of strict implication. When Wittgenstein said that all logical (necessary) propositions say the

1. Lewis and Langford, *Symbolic Logic*, p. 163.

same thing, namely, nothing, he meant that logical propositions say nothing about the realm of facts. Hence the question whether they are true or false does not arise. All necessary propositions are explications of the meanings of propositions. They analyse their meanings and determine their essences (as opposed to accidental properties). It is an accident that a proposition, e.g., 'This cloth is red', chosen at random should be true or false. But the necessary proposition, 'A triangle is a figure bounded by three straight lines', expresses what is essential to itself.

When it is said that a necessary proposition is strictly implied by any proposition, this sense of implication is wholly different from the ordinary sense of implication. In the system of strict implication, a necessary proposition, e.g., '2+2 = 4,' is implied by any proposition, say, 'roses are red'. But this is not the sense in which ' this book is coloured' is implied by 'this book is red'. From the proposition 'this book is red', we can deduce the proposition 'this book is coloured'; because there is an intrinsic connection between their meanings. But from the proposition 'roses are red' we cannot deduce '2+2 = 4', because there is no connection between their meanings. Deduction is in virtue of the connection of meanings between propositions, but strict implication may hold regardless of any such connection. Hence valid deduction is not possible in every case of strict implication.

Necessities and impossibilites are not truth-values of propositions, but what might be called form-values. A proposition is necessary if its defferent elements form a consistent whole; it is impossible if its different elements contradict one another. The system of strict implication contains such propositions as :

"'*p* is necessary' strictly implies '*p* is true'"

and "'*p* is impossible' strictly implies '*p* is false'".

These propositions owe their plausibility to the ambiguity of the words 'true' and 'false'. The sense in which a necessary or analytic proposition '3×4 = 12' is true, is different from the sense in which a material or synthetic proposition, 'this cloth is red', is true. We have, to avoid this ambiguity, already stated that analytic propositions should not be referred to as true but self-consistent. Likewise, impossible propositions should not be referred to as false but self-contradictory.

If in the definition of strict implication we interpret the notion of impossibility intensionally, it asserts an analytic proposition. If

it is impossible that p is true and q false in virtue of the connexion of meanings between them, then $p \prec q$ is an analytic proposition. In such a case p and q together form a consistent whole. The connexion betwecn p and q will be a necessary one. And we hold that all implications express necessary connexion. But unfortunately Lewis has not used the notion of 'impossibility' intensionally. For him, as already stated, if p is an impossible proposition *by itself*, $p \prec q$ holds.

The reason for rejecting material implication as the basis of deduction is that it is defined in terms of extensional disjunction. $p \supset q. = .\sim p \vee q$, i.e., 'either p is false or q is true'. Extensionl disjunction is not a necessary relation between two propositions. $\sim p \vee q$ may be held on the basis of truth-value of p itself or of q itself separately, though there may not be any necessary connexion between their meanings. It may hold between propositions which are independent of one another. Implication, expressing a necessary connexion, cannot hold between propositions which are independent of one another. The members of a proposition asserting implication may be analytic or synthetic propositions, but the proposition must be analytic. Consider the following examples ;

(1) " '2+3 = 5' implies '2+4 = 6' "

(2) " 'This cloth is red' implies 'this cloth is coloured' "

In (1) the members are analytic propositions and in (2) the members are synthetic propositions, but both the propositions (1) & (2) are analytic. To know that " '2+3 = 5' implies '2+4 = 6' ", and " 'this cloth is red' implies 'this cloth is coloured' ", we need not refer to any facts, but simply to analyse the meaning of the implier.

Lewis falls into the trap of extensional logic when he interprets 'it is impossible that p is true and q is false' as implying that 'if p is an impossible proposition by itself (without any connexion of meaning with q), it strictly implies q. Similarly, his definition of 'p and q are consistent' as 'it is possible that p and q both be true' is also extensional. Suppose that p is impossible, then p and any proposition q cannot both be true. Hence, according to this definition, p and q are inconsistent, regardless of the meaning of q. The objection to this definition is based on the fact that 'both p and q are true' may not be possible because of the nature of p itself or of q itself separately, though not because of any relation between their meanings.

Lewis seems to have changed his position in *Symbolic Logic* (p.154) when he says : "Any two propositions,both of which are true, must be consistent. This implication is not reversible : two propositions may be consistent though one is false and the other true, or though both are false". But if any two propositions which are true are consistent without any relation between their meanings, this is also an *extensional* notion of consistency.

In his article 'The meaning of Implication', Mind, 1936, Bronstein tries to remove some misunderstanding regarding the paradoxes of strict implication by distinguishing between 'implies' and 'implicates'. The following quotation states the distinction :

"I. If p implies q in virtue of what p asserts, this shall be symbolised by

$$p \rightarrow q$$

and is to be read 'p implies q'.

II. If p implies q (1) in virtue of the fact that p is a proposition, regardless of what it asserts (in which case q will be a necessary proposition), or (2) in virtue of the fact that q is a proposition, regardless of what it asserts (in which case p will be an impossible proposition), this should be symbolised by

$$p \rightarrowtail q$$

and is to be read 'p implicates q'. In either case p will be called the implicator and q the implicand. Propositions that are either necessary or impossible are second-order propositions; first-order propositions will be called 'material propositions'. The difference between a first-order proposition and a second-order proposition is that the former says something *about the realm of facts*, while the latter says something about 'propositions'."[1]

Bronstein gives 'A proposition is either true or false' as an example of a necessary proposition. It states a property (as opposed to accidents) of a proposition. A property is a necessary condition of a 'proposition'.

The following examples clarify the distinction between 'implies' and 'implicates' :

(1) 'Dante assigned all innocent pagans to limbo' *implies* 'Dante assigned Plato to limbo'.

1. Bronstein, *loc. cit.*, p. 171.

(2) 'It is false that no propositions are both true and false' *implicates* 'Dante assigned Plato to limbo.'

(3) 'Caesar Crossed the Rubicon' *implicates* 'The Law of Contradiction'.

According to Bronstein (2) & (3) are cases of 'implicates' and hold not in virtue of what the propositions 'Dante assigned Plato to limbo' and 'Caesar crossed the Rubicon' mean or assert, but in virtue of the fact that they are propositions.

Any case of strict implication is either a case of 'implies' or of 'implicates'. If it is a case of 'implies', truth-values of propositions are irrelevant; the relation of 'implies' can hold in virtue of the relation between the meanings of the implicator and the implicand, and such a relation is ordinary implication. If it is a case of 'implicates', it is certainly not ordinary implication. It cannot certainly be maintained that the law of contradiction can be deduced from or implied by any proposition, say, 'Caesar crossed the Rubicon'.

In an article entitled 'Facts, Propositions and Entailment' in Mind, 1948, A.N. Prior also puts forward views which may be interpreted to justify the paradox that 'a necessary proposition is strictly implied by any proposition'. He holds that any proposition asserts (at least implicitly) something about all objects whatsoever. 'This table is brown', for example, says among other things that it is not the case that the table is non-brown and roses are red; and so on. Prior observes, "The mystery of this sort of entailment, if mystery it is, is simply the ancient mystery of negation. Negation, as Morgan and many others have seen, always divides the universe in two; always involves a reference to 'the rest of the universe'; and since 'all determination is negation', every fact involves such a reference to 'the rest of the universe', and can be expressed in a negative form which makes this reference explicit."[1] On the basis of it, it has been argued that "there is a connexion of meanings between any two propositions, and a necessary or impossible proposition has with any other proposition a connexion of meanings such as will validate one or other of the paradoxical inferences discovered by Lewis."[2]

1. A.N. Prior, *loc. cit.*, p. 67.
2. Jonathan Benett, 'Meaning and Implication', *Mind*, 1954, p. 462.

On examining Prior's views it appears that what he means is that the law of identity and the law of contradiction are implied by every proposition. Any proposition implies the negation of many other propositions and the denial of its contradictory proposition. But here again we must distinguish between universal and necessary assumptions (or conditions) of thought and implication. The law of identity and the law of contradiction are universal and necessary assumptions or conditions of all our thinking, but they cannot be said to be implied by every proposition because implication ordinarily refers to the relation among propositions all of which are within the sphere of these laws and have a definite and specific connexion of meanings. From the impossible proposition which denies the law of contradiction we cannot deduce any proposition.

The way or the sense in which a proposition negatively refers to every other proposition is not the sense in which it is related to the implied proposition.

In Indian philosophy, Jainism seems to hold a view somewhat similar to that of Prior. According to Jainism, everything possesses innumerable, positive and negative characters by which it is related to every other thing of the universe; therefore, the knowledge which we get about any thing is also relative (Syādvāda). On this view any proposition about a thing is related to every other proposition; if we know one thing completely, we know all things completely.[1] But Jainism has not attempted to show the logical interconnexions between all things or propositions of the universe as it is an impossible task. And unless this is done, the theory that a proposition is related to every other proposition must be rejected as mere dogma. Where Jainism refers to absolute knowledge, it must be taken in the sense of intuitive knowledge of a Jina. But intuitive knowledge is not logical, though it may be supralogical. We are concerned with the logic of man and not with the logic of superman.

We hold that the paradoxes of strict implication cannot be justified, because a necessary or impossible proposition has no intrinsic connexion of meaning with each and every proposition of the universe. But it may be argued that a proposition in a system is paradoxical only when it is judged from the point of view of another system. We do not deny that in Lewis's system there

1. Eko bhāvah sarvathā yena drstah sarve bhāvāh sarvathā tena drstāh.

exists the propositional relation which is symbolised by '⥽', and that he has tried to use it consistently. But his claim to identify this relation with ordinary implication cannot be accepted. It is the ideas expressed in the following quotation which we have tried to repudiate in this chapter:

"In the light of all these facts it appears that the relation of strict implication expresses precisely that relation which holds when valid deduction is possible, and fails to hold when valid deduction is not impossible. In that sense, the system of Strict Implication may be said to provide that canon and critique of deductive inference which is the desideratum of logical investigation."[1]

1. Lewis and Langford, *Symbolic Logic*, p. 247.

7

Implication and Meaning

In his article entitled 'Intensional Relations', published in *Mind*, 1930, Everett J. Nelson has criticised Lewis's paradoxes of strict implication and presented a concept of implication based on *connexion of meanings*. Lewis, according to Nelson, has introduced into his system the intensional notion of possibility, and its derivates, but he uses it only as a function of purely extensional relations. Instead of interpreting the disjunctive proposition '*p* or *q*' as the proposition that a disjunctive relation necessarily holds between its two members by virtue of the necessary connexion between their meanings, he has interpreted it as the proposition that it is necessary that one of its members be true, whether because of an intensional relation between them or because one of them is true as a matter of fact.

Nelson outlines a system of logic in which the relations of 'entailment' (which term he uses for implication as ordinarily understood), consistency, equivalence, contradiction, disjunction and conjunction are determined by the meanings of the propositions in relation to one another, and not by their truth-values. The system is based on the notion of consistency symbolised by '$\bigcirc$' which he explains as the relation which holds between a pair of propositions when there is nothing in their meanings such that the truth of one necessitates the falsity of the other. He rejects the definitions of consistency given by Lewis and Russell as being extensional.

Lewis defines "*p* and *q* are consistent" as "it is possible that *p* and *q* both be true."[1] Suppose that *p* is impossible, then *p* and any proposition *q* cannot both be true. Hence, *p* and *q* are inconsistent, irrespective of the meaning of *q*. If we substitute *p* for *q*, we

1. *A Survey of Symbolic Logic*, p. 293.

find that *p* is inconsistent with itself. Nelson's objection to this definition is based on the fact that "both *p* and *q* are true" may not be possible because of the nature of *p* or of *q* itself separately, and not because of any relation between their meanings.

Russell defines "*p* and *q* are inconsistent" as "either *p* or *q* is false". According to this definition, if *p* is a false proposition, it is inconsistent with any proposition, even with itself. "To define", says Nelson, "all logical notions extensionally would seem to imply the view that the inconsistency (or logical opposition) of propositions is a relation not intrinsic to the proposition, but is rather founded on their truth-value; and hence that knowledge of the consistency or inconsistency of any given contingent propositions cannot arise through an examination of the propositions themselves but presupposes a knowledge of their truth-values. Now this view, in holding that inconsistency is a truth-function, puts the cart before the horse. For if two propositions be incompatible, then I can conclude that not both are true (though I do not use it as a definition of consistency, this idea suggests Leibniz's notion of compossible). But from the mere data that one of them is false, I cannot conclude that they are incompatible. Such definitions of logical notions as this of Russell are in terms of matter of fact, not of logical necessity."[1]

Nelson's criticisms against the extensional notion of consistency and inconsistency seem valid, but his own concept of these relations is not very clear and satisfactory. We shall discuss it while dealing with his views on entailment. What we consider to be the main criticism against the extensional definition of consistency and inconsistency is that it establishes these relations between propositions which are independent of one another. It is possible for both the propositions–'roses are red' and '2+2 =4' — to be true. Hence, in accordance with Lewis's definition, these are consistent. But each is true by itself and not due to any connexion of meanings between them. The meaning of one is not determined by the other. Each belongs to a different sphere of meaning. The question of consistency or inconsistency arises only with regard to those propositions which belong to the same sphere of meaning. No doubt when two propositions are consistent, it is possible for both of them to be true. But the possibility of both being true is not the

1. Nelson, *loc. cit.*, p. 441.

ground for their consistency. Consistency is an intrinsic relation between the propositions and does not depend upon their truth-values which are extrinsic.

Lewis has not clarified the meaning of the word 'possible' in his definition of consistency. In the absence of any clarification of this word and considering that the word 'impossible' in the definition of strict implication has been interpreted extensionally,[1] justifying the paradoxes, we cannot but interpret the proposition 'it is possible that *p* and *q* both be true' as 'it is possible that *p* be true by itself and *q* be true by itself.' This is the extensional interpretation of 'possibility'; there is no connexion of meaning between *p* and *q*. There is, however, another sense in which the word 'possible' may be used. We may say that 'it is possible that both *p* and *q* be true because both are the *constituents of the same coherent system* which has internal relations among its different constituents, and thus have intrinsic connexion of meanings between them. There is nothing in their meanings such that the truth of one necessitates the falsity of the other. In other words, they are consistent. We hold that this is the correct intensional interpretation of consistency. The correct intensional definition of '*p* and *q* are inconsistent' is that 'it is impossible for both *p* and *q* to be true because their meanings are contradictory.' In other words, the truth of one necessitates the falsity of the other. The impossibility of both *p* and *q* being true is the impossibility of both forming or becoming constituents of the same coherent system. But the question of forming a particular coherent system arises only with regard to those propositions which have the same sphere of meaning. What we mean by the same sphere of meaning can be understood by an example. The propositions — 'three angles of a triangle are equal to two right angles' and 'a triangle is a figure bounded by three straight lines' — belong to the same sphere of meaning, but the propositions 'roses are red' and 'a triangle is a figure bounded by three straight lines' belong to different spheres of meaning, When the two propositions belong to the same sphere of meaning, the meaning of one affects and determines the meaning of the other; but when the propositions belong to different spheres of meaning, the meaning of one does not affect the other and the question of their consistency or inconsistency does not arise. The propositions of the latter type are independent of one another.

1. See *above*, Chap. VI.

The intensional definition of 'inconsistency'(i.e. of impossibility) which we have given, if applied to Lewis's definition of strict implication, avoids the paradoxes which arise by defining it extensionally.

We have seen that according to Russell's definition of inconsistency, a false proposition is inconsistent with any proposition even with itself. But we hold that the question of inconsistency between a false proposition and any other proposition chosen at random does not arise. It can arise only with regard to those propositions which fall within the same sphere of meaning to which the false proposition belongs. Even if the false proposition and any other proposition are within the same sphere of meaning, the relation of inconsistency solely depends upon the connexion of meanings between them. The falsity of a proposition is not the reason for its being inconsistent with any other proposition, but inconsistency between any two propositions is the reason for one of them to be false, provided we know the other to be true.

Now we shall discuss Nelson's notion of entailment which he defines in terms of consistency. He defines '*p* entails *q*' as '*p* is inconsistent with the propositional function that is the proper contradictory of *q* (i.e., with$\sim q$)'.[1] Symbolically it can be expressed as :

$$p \mathrm{E} q = \sim(p \bigcirc \sim q)$$

By '*p*' Nelson does not mean '*p* is true' but simply the propositoinal function itself, and by the term propositional function he means any form such that when the values of the variables in it are determined the result is a proposition. He takes propositional function rather than proposition as fundamental because intrinsic logical relations hold directly between the structures or formal significances of propositions. '$\sim q$' does not mean '*q* is false' but symbolises the propositional function which is the proper contradictory of the propositional function symbolised by '*q*'. Entailment, says Nelson, not being defined in terms of truth-values, is a necessary connexion between meanings.

As Nelson's definition of entailment is in terms of propositional functions, we must first attempt to understand the nature of a propositional function. In the most general sense, every expression which contains two or more variables may be called a function,

1. *loc. cit.*, p. 445.

and a variable is a symbol which denotes any one of a class of elements. But Russell seems to have introduced the specific form of function called propositional function in order to facilitate the analysis of general proposition. What is asserted in the general proposition "All men are mortal" is that 'any' man taken from the class 'men' is a mortal. If x represents 'any' man, then we get the assertion: "If x is a man, then x is mortal, for all values of x". The expression "If x is a man, then x is a mortal" is a propositional function. Russell brings out the use of propositional functions clearly in the following passage:

"Now when you come to ask what really is asserted in a general proposition, such as 'All Greeks are men' for instance, you find that what is asserted is the truth of all values of what I call a propositional function. A propositional function is simply any expression containing an undetermined constituent, or several undetermined constituents, and becoming a proposition as soon as the undetermined constituents are determined. If I say 'x is a man', or 'n is a number', that is a propositional function."[1]

It is clear that propositions are obtained from propositional functions by substituting determinate values or individual constants for the individual variables occurring in them. The process of obtaining a proposition from a propositional function by substituting determinate values for variables is called *instantiation.* A propositional function is neither true nor false for it does not refer to any determinate fact. It is only a proposition which is true or false. The propositional function 'x is a man' is neither true nor false, but if we substitute 'Johnson' for 'x', we get the proposition 'Johnson is a man' which is true. Russell brings out the distinction between a propositional function and a proposition in the following lines:

"By a 'propositional function' we mean something which contains a variable x, and expresses a *proposition* as soon as a value is assigned to x. That is to say, it differs from a proposition solely by the fact that it is ambiguous: it contains a variable of which the value is unassigned. It agrees with the ordinary functions of mathematics in the fact of containing an unassigned variable; where it differs is in the fact that the values of the function are propositions."[2]

1. *The Monist*, 1919, p. 192.
2. *Principia Mathematica*, Vol. I, p. 38.

Nelson defines entailment in terms of propositional functions because he holds that intrinsic logical relations (and entailment, according to him, is such a relation) hold directly between the structures or formal significance of propositions. So, according to him, the main use of a propositional function is that it exhibits the structure or formal significances of a proposition. To understand this point clearly we must first understand the distinction between the form and constituents of a proposition. The distinction has been traditionally expressed as a distinction between form and matter. The traditional logicians did not, however, state the distinction clearly because they did not carry the analysis far enough. According to them, every proposition can be regarded as asserting a relation between classes, and, therefore, there are four, and only four kinds of propositions ;

A. All X is Y. Universal affirmative,
E. No X is Y. Universal negative,
I. Some X is Y. Particular affirmative,
O. Some X is not Y. Particular negative.

This schedule of propositions is based on the fact that we can assert of the class X that it is either wholly, or partially, included in, or excluded from the class Y. Though four kinds of propositions were recognized, the traditional logicians regarded every proposition of the same logical form — subject-copula-predicate. This view is however rejected by the modern logicians.

The distinction between constituents and form can be easily understood by examples.

Let us consider the following set of propositions:

1. Johnson is tough.
2. Indira Gandhi is intelligent.
3. Reagan is wise.

It is obvious that these three propositions are all of the same form, but no two have the same constituents. They are about different subjects.

Let us now consider the following set of propositions :

1. Rāma was fortunate.
2. Rāma married Sitā.
3. Rāma was a king.

These three propositions have a common constituent, viz. Rāma, but they are of different forms. Their forms are subject-predicate, relational and class-membership respectively.

Let us now take the following set of propositions :

1. Rāma killed Rāvaṇa.
2. Krishṇa killed Kaṁsa.
3. Lakshmaṇa killed Meghnātha.

The form of all these three propositions is the same, but the constituents are different. We see, to quote Russell, that 'the form remains unchanged throughout the series, but all the constituents are altered. Thus form is not another constituent, but is the way the constituents are put together.'[1]

A proposition consists of two kinds of constituents : particular and universal. A *particular* is a constituent which can occur in a proposition only as an individual occurs. A *universal* is a constituent which can occur in a proposition in such a way as to combine the other constituents into a unity. In the proposition 'Rāma loves Sītā', 'loves' combines the two constituents 'Rāma' and 'Sītā' into a unity; similarly, in the proposition,'Ramesh gave Mohan a book', 'gave' combines the three constituents—Ramesh, Mohan and book—into a unity. Thus, 'loves' and 'gave' are universals, while 'Rāma', 'Sītā', 'Ramesh', 'Mohan' and book' are particulars. In a proposition there must be at least one universal, that is a combining constituent. A particular can be combined but it cannot perform the function of combining. A universal cannot be replaced by an individual. There are propositions in which all the constituents may be universals, e.g., 'honesty differs from falsehood', but even in such a case one constituent performs the function of combining the other constituents into a unity. Thus we may distinguish between two kinds of universals—(1) the principal[2] universal which performs the task of combining and cannot be replaced by an individual, and (2) the simple[3] universal which simply occurs in a proposition and cannot be replaced by an individual, but has no combining function, e.g., 'honesty' and 'falsehood'.

1. *Our Knowledge of the External World* (1926), p. 43.
2. Chadwick has used this word for this type of universals.
3. I am using this word for the second type of universals.

Let us now examine Nelson's definition of entailment in terms of propositional functions. The definition suggests that entailment holds by virtue of the formal meanings of the propositions concerned. The formal meaning of a proposition is determined by its universal terms. Now the question before us is what the utility and function of particular terms are. Are they totally irrelevant to the determination of the relation of entailment ? The universal terms only determine the form or formal meaning of a proposition, but the formal meaning is not the whole meaning of a proposition. The whole meaning of a proposition is determined by both particular and universal terms. Universal terms determine a wider class of thoughts or concepts as its meaning; particular terms confine that meaning to a particular thought. The real meaning of a proposition is always a *particular thought* consisting of both universal and particular terms.[1] In the absence of particulars, form would be mere abstraction and would not convey any meaning. 'A loves B' does not express any meaning unless we assign determinate values, which are particulars or individuals, to A and B. It is these values which determine the meaning of a proposition as a particular thought. 'A loves B' is not a proposition and cannot entail any other proposition, because only propositions, and not propositional functions, can entail other propositions. Propositional functions are mere empty sounds unless values are assigned to the variables. They may be *linguistic devices* to express the form of a proposition or of a class of propositions, but are certainly not the whole meaning of a proposition. Hence Nelson's definition of entailment in terms of propositional functions is unsatisfactory; entailment holds between propositions or between their meanings, and refers to a *particular coherent system of thought*.

Moreover, there are propositions which have no corresponding propositional functions, because they contain only universal terms and no particulars which can be expressed by variables. For example, the propositions '$1+1 = 2$', '$x+3 = 4$', and 'honesty is the best policy', have no corresponding propositional functions. Even the symbol 'x' in the proposition '$x+3 = 4$' has a determinate value '1' and cannot be replaced by any other number. If we substitute any other number, the whole meaning of the proposition

1. Even where all the terms are universals—principal and simple—the meaning expressed is a particular thought.

will be changed, and the relation of 'is equal to', '=', will not hold between the two sides of the proposition. The proposition '1+1= 2' implies '1+2 = 3', but we cannot express it in terms of propositional functions.

Nelson seems to have been led to introduce the notion of propositional function in his definition of entailment due to the influence of Russell who holds that a general proposition, e.g., 'all men are mortal' expresses a formal implication and can be analysed as 'if x is a man, then x is mortal, for all values of x'. Or, to use Russell's language of material implication, "for every value of x, the proposition 'x is a man' materially implies 'x is mortal' " states a formal implication. But the use of the words 'materially implies' or 'if .. . then' in this analysis of a general proposition is misleading and does not correspond to ordinary implication. 'x is a man' is not a proposition and does not imply anything. 'If x is a man, then x is mortal, for all values of x' is a simple proposition and not a compound proposition asserting the relation of implication. It does not assert the relation of implication between two propositions, but asserts the invariable relation *between two terms*—'man' and 'mortality'. A propositional function may facilitate the analysis of a general proposition, but is not itself a proposition, and, therefore, cannot imply any other proposition. The purpose of a propositional function may be to exhibit the logical form of a proposition. But the logical form itself is not the proposition. A proposition consists of both form and matter. In the argument 'if A is greater than B and B is greater than C, then A is greater than C', the words 'if ......then' express only the property of transitivity of the logical relation 'is greater than' and not any case of implication, for the argument[1] consists of propositional functions and not propositions.

Let us now consider Nelson's notion of consistency or inconsistency which occurs in his definition of entailment. It is obvious that he interprets this notion intensionally, that is, in terms of the meanings of the propositions concerned. But the notion of propositional function being used in the definition of entailment, it

1. In chapter II we have given arguments of this type as examples of implication. But they should be taken as examples of the *logical form* of implication and *not instances* of implication. They become instances of implication only when determinate values are assigned to variables.

appears that what he means by the meaning of a proposition is its formal meaning. So his definition of '*p* entails *q*' may be translated as 'the formal meaning of *p* is inconsistent with the formal meaning of $\sim q$'. But we have already pointed out that the formal meaning of a proposition apart from its matter or particular constituents is mere abstraction and does not convey any meaning unless applied to a particular. Form without matter is empty and matter without form is blind. Both form and matter are necessary to determine any real meaning which is always a *concrete whole*. Inconsistency is always inconsistency between a concrete whole and its parts or between its different elements. We cannot think of inconsistency apart from a concrete whole. A connexion of meanings is possible only between a whole particular system and its elements or between its different elements. As Nelson was mistaken on the meaning of a proposition, he could not clarify correctly what is meant by the 'connexion of meanings' between propositions though maintaining that entailment holds by virtue of it.

In his article 'Is Strict Implication the Same as Entailment',[1] A. E. Duncan Jones adopts a system similar to Nelson's. Both maintain that when one proposition entails another there must be some connexion of meanings between them. Duncan Jones describes entailment as 'a relation which holds between two propositions *p* and *q*, only when *q* arises out of the meaning of *p*', and attempts a series of definitions which do not seem to add anything by way of exhaustiveness and precision to Nelson's. What both mean by connexion of meaning between *p* and *q* seems to be that what *q* says is at least part of what *p* says. Nelson says, 'Entailment is, an identity between a structural part (though not necessarily less than the whole) of the antecedent and the entire consequent."[2] Jonathan Bennett holds that this suggests that 'where *p* E *q*, *p* is equivalent to *q* plus some other proposition *r*, which means that any proposition asserting an entailment is capable of statement in the form '*pq* E *p*'.[3] Nelson denies that *pq* E *p*. I think the confusion is because he has not clearly explained what he meant by the consequent being a structural part of the antecedent. He could certainly not have meant that *p* is equivalent to *q*

1. *Analysis*, 1934-35, pp. 70-80.
2. 'Intensional Relations', *Mind*, 1930, p. 447.
3. 'Meaning and Implication', *Mind*, 1954, p. 458.

'plus' some other proposition *r*, because the word 'plus' suggests that *p* is a mathematical sum of certain propositions. There is a difference between a structural whole and a mathematical sum. The concept of 'structure' (used by Nelson) suggests that its parts form an organic whole and affect each other, while the concept of 'mathematical sum' suggests that its different members are just put together without any interrelation between each other.

Another meaning which Nelson's view quoted above suggests is that entailment may hold even when the whole antecedent is identical with the entire consequent. But he seems to be mistaken here. Entailment is a relation between two propositions which are *partly distinct* from each other, while identity is not a relation at all. A relation holds between two things which are at least partly distinct, but in identity there is only one thing and one alone. When we say *p* is identical with *q*, *p* and *q* are not two propositions but one and the same proposition, though designated by different symbols; we simply say the same thing again. Nelson holds that 'intensional equivalence is what is usually called the relation of identity.' According to him, every case of identity is a case of entailment. He, therefore, gives the following definition of extensional equivalence :

$$p = q. = . p \mathrm{E}\, q. q \mathrm{E}\, p$$[1]

In accordance with this definition every proposition, being identical with itself, entails itself, $p \mathrm{E}\, p$. This assertion, together with $p \mathrm{E}\, q. \mathrm{E} . p \bigcirc q$, gives rise to $p \bigcirc p$. A proposition is consistent with itself. But here again Nelson is mistaken. A proposition is not consistent but identical with itself. Consistency is a relation between two *partly distinct* propositions, while identity is not a relation at all. Two propositions are said to be consistent when both belong to the same sphere of meaning, and there is nothing in the meaning of one which contradicts anything in the meaning of the other. While it is meaningless to say that a proposition as a whole is consistent with itself, we may say that a proposition is self-consistent which means that its *different elements* do not contradict each other. A proposition is said to be self-contradictory when its different elements contradict each other. But a proposition *as a whole* does not contradict itself or is not inconsistent with itself.

1. *Loc. cit.*, p. 446.

The significance of this view is evident when we recall that according to Russell a false proposition is imcompatible with itself, and according to Lewis an impossible proposition is inconsistent with itself. Another significant point which follows from our view is that a proposition, being identical with itself and inconsistent with its contradictory, does not entail its contradictory, $\sim(p \text{ E} \sim p)$, which is interesting in reference to systems in which a false or an impossible proposition implies its contradictory.

Nelson discusses certain characteristically extensional propositions which have no corresponding form in intension. He takes two impartant extensional assertions, '$p \supset p \vee q$' and '$pq \supset p$', and show that they have no corresponding propositional functions in his intensional system. It is needless to say that the so-called paradoxes of implication arise due to such extensional assertions and by denying them he avoids the paradoxical theorems in his system.

Let us first discuss the assertion $p \supset p \vee q$, that is, '*p*' materially implies '*p* or *q*'. In chapter V we have distinguished between extensional and intensional disjunctions. An extensional disjunction is satisfied merely if one of the disjoined propositions is true. To decide whether an extensional disjunction holds we have to know on the basis of facts whether any of the disjoined propositions is true, and if any one of them is true, the truth or falsity of the other is irrelevant as there is no necessary connexion between their truth-values. But an intensional disjunction between *p* and *q* holds only when there is a necessary connexion of meanings between them, and we can know it by mere formal analysis of our thought without reference to facts.

No doubt we can assert that "if '*p*' is true, then 'either *p* or *q*' is true", and such an assertion is quite consistent with an extensional system in which it is used. Our objection is only this that the use of the word 'implication' or 'if......then' here is unwarranted as the notion of material implication is different from the ordinary sense of implication when we say that in any deductive system the premises imply the conclusion. Russell crossed the boundary line of his own system and used this notion in the ordinary sense.[1]

The critic may ask that if the relation between '*p* is true' and 'either *p* or *q* is true' is not that of implication, how it is that

1. See *Principia Mathematica*, Vol. I, pp. 90 and 94.

we pass from one to the other. I think we do not pass from '*p*' to '*p* v *q*' in the sense of inferring the latter from the former. What happens is this that *p* is a member of an extensionally disjunctive proposition '*p* v *q*'. So if we assert *p*, we can also assert '*p* v *q*' because *q*, being only externally connected with *p*, is irrelevant to the assertion of '*p* v *q*'. *q* being irrelevant, we can replace '*p* v *q*' by any other proposition, say, '*p* v *s*', '*p* v *t*', etc. In '*p* v *q*' there is simply a *reassertion* of *p*. It being a reassertion of *p* only we *cannot* say that we pass from '*p*' to '*p* v *q*', or that we infer the latter from the former. In 'passing' or 'inferring' there are *two elements* necessary which are *partly* distinct from but internally related to each other.

The other important extensional assertion which Nelson avoids in his system is '$pq \supset p$'. We must here also distinguish between the extensional and intensional notions. In intensional conjunction of *p* and *q* both have a connexion of meaning and must function as a unity, while in extensional conjunction both *p* and *q* are independent of each other, but are just put together and have no other relation except that of being put together. Extensional conjunction is like a mathematical sum in which the different members are not modified but keep their identity. It is true that 'if *p* is true and *q* is true, then *p* is true'. But here again 'if . . . then' does not express any relation of implication. Nelson says, "Though '*p* and *q* entail *p*' cannot be asserted on logical grounds, I do not deny that from '*p* is true and *q* is true' we can pass to '*p* is true'. All I deny is that such a passage is in virtue of an entailment relation holding between '*p* is true and *q* is true' and '*p* is true'."[1] To a critic's querry as to the nature of this 'passing to' no successful answer seems to have been given. D. J. Bronstein says, "If the inference (for it is an inference) is not valid in virtue of entailment, in virtue of what is it valid ? What does the 'If . . . then' mean if it does not mean entailment."[2] I think that the confusion on this point is, as before, due to the wrong use of the words 'if . . . then' and 'passing to'. We do not pass from '*p* is true and *q* is true' to '*p* is true' nor do we infer the latter from the former. The proposition '*p* is true' is a member of the conjunctive proposition '*p* is true and *q* is true' whose members are externally related to

1. 'Intensional Relations', *Mind*, 1930, p. 448.
2. 'The Meaning of Implication', *Mind*, 1936, p. 166.

each other. So when we say that 'if p is true and q is true, then p is true', we simply *reassert* a member-proposition of the (extensionally) conjunctive proposition. No inference or implication is involved in reasserting a proposition. Reasserting is only saying the same thing again. There is some difference of time between asserting and reasserting a proposition, but the meaning of the proposition remains the same. Between asserting and reasserting some *time passes*, but in inference *we pass from one meaning to some other partly different meaning.*

Nelson's argument for denying pq E p in his system is as follows: "In view of the fact that a conjunction must function as unity it cannot be asserted that the conjunction of p and q *entails* p, for q may be totally irrelevant to and independent of p, in which case p and q do not entail p, but it is only p that entails p. I can see no reason for saying that p and q entail p, when p alone does and q is irrelevant, and hence does not function as a premise in the entailing. pq E p looks innocent enough, but in order to make explicit its real import, let us substitute p for r in the Principle of the Antilogism, (pq E $r. = .p{\sim}r$ E ${\sim}q. = .q{\sim}r$ E ${\sim}p$), we get :

$$pq \text{ E } p. = .p{\sim}p \text{ E } {\sim}q. = .q{\sim}p \text{ E } {\sim}p,$$ in which pq E p is equated to $p{\sim}p$ E ${\sim}q$. Therefore, the assertion of pq E p gives rise to an assertion in which the contradictory of what originally was irrelevant has become one of the complete terms of the entailment relation, in which entailment relation the antecedent and the consequent are logically independent of one another."[1]

Against this view we have already stated that p does not entail p, but is simply identical with itself. Hence p is not entailed by the conjunctive proposition pq, but is identical with one of its members. The innocent assertion pq E p is due to another innocent assertion p E p. Nelson is a victim of the latter. If we realise that p E p cannot be asserted, we will not commit the mistake of asserting pq E p. The failure to see that p does not entail but is identical with p, has forced Nelson to modify the principle of Syllogism. For if, in

$$p \text{ E } q.q \text{ E } r : \text{E} : p \text{ E } r$$

we substitute p for q, we obtain

$$p \text{ E } p.p \text{ E } r : \text{E} : p \text{ E } r$$

1. 'Intensional Relations', *Mind*, 1930, pp. 447-48.

which is of the form pq E p which he wishes to deny. **Hence** the Principle of Syllogism is restated in this way :

$$p \neq q \neq r : \mathrm{E} : p \mathrm{~E~} q . q \mathrm{~E~} r : \mathrm{E} : p \mathrm{~E~} r$$

in which

$$p \neq q \neq r . = . \sim(p=q) . \sim(p=r) . \sim(q=r)$$

But if it were realised that a proposition does not entail itself, there would not have been any need to state the rider, for then we could not substitute p for q in the Syllogism as it gives pEp, which we have already denied, as one of the premises. No doubt the Principle of Syllogism holds only when $p \neq q \neq r$, but the reason for this is that a proposition does not entail itself. If the Principle of Syllogism is true only when $p \neq q \neq r$, then the Principle of Syllogism entails $p \neq q \neq r$. Hence Nelson should have written the converse of what he did write, viz. :

$$(p \mathrm{~E~} q . q \mathrm{~E~} r : \mathrm{E} : p \mathrm{~E~} r) : \mathrm{E} : (p \neq q \neq r)$$

8

Implication and Facts

Does implication express relation beween facts, and, if so, what is that relation? It is this question which Ewing asks in somewhat different words, 'Can I by inference pass with logical necessity from one fact to another, different fact, or is the conclusion of any inference . . . always merely part (or all) of the premises restated in different language?'[1] Philosophers have discussed and expressed different views about it. For example, Wittgenstein says, 'If *p* follows from *q*, the sense of *p* is contained in the sense of *q*'[2]. And he also said, 'If *p* follows from *q* and *q* from *p*, then they are one and the same proposition.'[3] As against this Moore says, '. . . I do not see how we can possibly do justice to the facts without supposing that there are genuinely different propositions and genuinely different facts, which nevertheless mutually entail one another.'[4] In his article 'The Nature of Entailment' published in *Mind*, July 1940, Norman Malcolm expresses some considerations in favour of the view of Wittgenstein, that when one proposition entails another, the fact expressed by the consequent is identical with that expressed by the antecedent, or at least is 'not a new fact in addition to' the first. This position was defended with some modification by A. N. Prior in his article 'Facts, Propositions and Entailment', *Mind*, 1948. E. Toms, criticises this and holds that 'factual content cannot be regarded as contributing either to the derivation of an entailment or to its

1. A.C. Ewing, *A short Commentary on Kant's Critique of Pure Reason* (1961), p. 20.
2. *Tractatus* (1961), 5.122, p. 75.
3. *Ibid.*, 5.141, p. 79,
4. 'Facts and propositions', *Proceedings of the Aristotelian Society*, Suppl. Vol. VII, p. 198.

definition.'[1] It holds in virtue of the formal structures of the constituent propositions.

The view that implication expresses a connexion between facts has its root in the belief that every proposition implies or expresses facts corresponding to it. To get rid of this belief we must distinguish clearly between the meaning of a proposition and the facts corresponding to it. The meaning of a proposition is independent of any fact. What a proposition expresses is its meaning. It does not express any fact. From the statement of a proposition we cannot know what the facts are. To know whether it expresses a fact or not, we have to adopt a different method of verification by sense-experience. Facts make a proposition true or false. If there are facts corresponding to a proposition, it is true; if there are no facts corresponding to it, it is false. Facts are neither true nor false. They simply are. A fact is what the case is. It is only propositions which are true or false.

Sometimes confusion arises between stating a proposition and asserting it. The statement of a proposition is different from its assertion. When I state the proposition that 'the Great Wall of China is 1,259 miles in length', everybody will understand its meaning,but to understand its meaning does not mean or imply that it is true. To know its truth, one has to verify it. But if I assert it, it means or implies that it is true. In ordinary language the linguistic form of stating and asserting a proposition may be the same. We can understand whether one is stating or asserting a proposition either by some other words which precede or follow it or by vocal emphasis. The propositions which follow the words 'if . . . then . . .' are stated, while the propositions which are preceded or followed by the words 'is true' are asserted. The assertion of a proposition is equivalent to a conjunctive proposition consisting of two propositions—one stating the proposition and the other stating its truth-value. When I assert the proposition that 'the Great Wall of China is 1,259 miles in length', it means that this proposition is true. Propositions which state truth-values of other propositions are second-order propositions.

When we state a proposition everybody understands its meaning without knowing anything about its truth-value. The

1. 'Facts and Entailment', *Mind*, 1948, p. 236.

meaning of a proposition is independent of its truth-value. But the question is what the meaning of a proposition is. What is it that everybody understands when a proposition is stated ? To this question we must concentrate our attention now. The question of the meaning of a proposition is discussed sufficiently in contemporary Philosophy. The authors[1] of 'The Meaning of Meaning' have elicitated sixteen main definitions of 'Meaning'. We shall state and examine them briefly . Meaning is—

1. An Intrinsic property.
2. A unique unanalysable Relation to other things.
3. The other words annexed to a word in the Dictionary.
4. The Connotation of a word.
5. An Essence.
6. An activity Projected into an object.
7. (a) An event intended.
   (b) A volition.
8. The place of anything in a system.
9. The practical consequences of a thing in our future experience.
10. The theoretical consquences involved in or implied by a statement.
11. Emotion aroused by anything.
12. That which is actually related to a sign by a chosen relation.
13. (a) The Mnemic effects of a stimulus. Associations acquired.
    (b) Some other occurrence to which the Mnemic effects of any occurrence are appropriate.
    (c) That which a sign is interpreted as being of.
    (d) What anything suggests.

In the case of Symbols.

That to which the user of a symbol actually refers.

14. That to which the user of a symbol Ought to be referring.
15. That to which the user of a symbol Believes himself to be referring.
16. That to which the Interpreter of a symbol
    (a) Refers.

1. C.K. Ogden and I.A. Richards.

(b) Believes himself to be referring.

(c) Believes the User to be referring.

Definition No.1 suggests that meaning is an intrinsic property of a proposition as length or breadth is an intrinsic property of an object. Propositions seem to possess a peculiar stuff which is meaning. But we do not know any details about the nature of this stuff. If meaning is a property just like other physical properties, we should be able to perceive it by the senses. But nobody perceives it, everybody simply *understands* it.

Def. 2 implies that a proposition is necessarily related to some other thing. Either the relation or the thing related may be taken as the meaning of a proposition. If meaning is a relation, what is its nature? Is it extrinsic, or intrinsic? A factual relation, e.g., 'A is greater than B', can be perceived by the senses, but we never perceive the relation of meaning. If the meaning of a proposition is the thing to which it is related, then every proposition expresses a thing or a fact because every proposition has a meaning. The logical positivist holds such a view. A proposition which does not express any fact is meaningless or pseudo. But the view that there are meaningless propositions has its roots in the dogma that "the 'meaning' of a proposition is verifiable by *sense-experience*" from which the conclusion is derived that those propositions which are not verifiable are meaningless. But to give a definition of a term and then to derive a conclusion from it, will neither justify the definition nor the conclusion unless the former is in accordance with the common and logical usage of the term. The correct position is that we understand the meaning of a proposition without reference to any fact. We even understand the meaning of metaphysical and ethical propositions which can never be verified by sense-experience.

Def. 3. Dictionary meaning is only linguistic translation. It is nothing more than substituting one symbol for some other. It is like a mathematical equation in which the symbols of both the sides express the same meaning. Its value is insofar as it helps us to understand the meaning of some unfamiliar or difficult symbol through some other. But symbols themselves are not meanings. They express meanings. What we need is the meaning of a symbol and not a symbolic translation.

Def. 4. The traditional logicians distinguished between the denotation and the connotation of a word. The denotation of a

term is the set of things to which it can be correctly applied, and the members of this set are said to be denoted by the word. The connotation of a word is the group of properties in virtue of which anything is a member of the set which is the denotation. The connotation of a word determines its denotation which in turn determines comprehension or connotation.

The denotation of a word being a fact or thing, we reject it as the meaning because, as we have already stated, we understand the meaning without reference to any fact or thing. The connotation or properties apart from things are abstract and mere nominal entities. We never experience and know properties apart from things. Properties are always properties *of* a thing.

Another defect in the traditional theory of connotation is the impossibility of applying it to proper names which obviously are the simplest symbols to convey meaning. Mill and Johnson hold that proper names are non-connotative, but the latter makes a reservation:

"This does not amount to saying that the proper name is non-significant or has no meaning; rather we find, negatively, that the proper name does not mean the same as anything that could be meant by a descriptive or connotative phrase; and positively that it does precisely mean what could be indicated by some appropriate descriptive phrase."[1]

In our opinion a proper name has a meaning but its meaning is a *set of thoughts* which can be expressed by some descriptive phrase also. Its meaning can be applied only to a *particular* thing. It denotes a thing only when the description *is applied* to that thing.

Def. 5. Essence, the meaning of the critical realists, may be regarded as connotation hypostatised, and taken as fact. The criticisms against fact as meaning have already been stated.

Def. 6. Schiller puts it as follows : "Meaning is an activity taken up towards objects and energetically projected into them like an '*a*' particle." This suggests psychological as well as factual activity towards objects. But such an activity is always subjective and personal. It differs from individual to individual. Hence, according to this definition, meaning would become subjective. Moreover, any factual activity refers to future. Therefore, we

1. *Logic*, Part I (1921), p. 96.

would not understand the full meaning of a proposition immediately but would have to wait for the future. Against this view, we hold that future action is a consequence of understanding the meaning of a proposition but is not itself its content. We understand the meaning or content of a proposition in the immediate present.

Def. 7. This definition is psychological. What the speaker intends to refer to are his own wishes or volitions and as such purely personal. If there is no objectivity of meaning in symbols, how can the listener understand the intentions of the speaker through them. Without accepting the objective character of symbols all communications become useless. A language transaction may be defined as a use of symbols in a way that thoughts occur in a hearer which are similar in all relevant aspects to those which occur in the speaker.

Def. 8. The meaning of anything is said to have been grasped when its place or significance in some system is understood. But meaning in this sense is too vague to be of much service. What is the nature of this system? If it is an objective system like a political, social or physical system, it is difficult to asceratin the *exact place* of anything in it immediately. But we understand the meaning of a proposition *immediately* without reference to any objective or factual system. Moreover, to ascertain the place of a proposition in a system we have to know its relation to other propositions. But the nature of these other propositions is not clear. A proposition is not an exclusive possession of any one system, but may belong to many systems. Hence, according to the definition we are considering, a proposition will not have one but many meanings, and this violates the ideal of communication that a proposition should convey the same meaning to everyone.

Def. 9. This definition is chiefly associated with the pragmatists. William James says : "The meaning of any proposition can always be brought down to some particular consequence in our future practical experience, whether passive or active."[1]

This suggests that the meaning refers to future and we cannot understand it immediately. Against such a view Lovejcy says, "Many of our meanings are retrospective. . . No logical hocus-

1. The Meaning of Truth, (1911), p. 210.

pocus can transubstantiate the meaning 'yesterday' into the meaning 'tomorrow'. . . It is, in very truth, a meaning intrinsically incapable of directly-experienced fulfilment......Without ever actually experiencing the fulfilment of these 'meanings', we nevertheless have an irresistible propensity to believe that some of them are in fact valid meanings. A judgment is its own master in deciding what it means, though not in deciding as to the fulfilment of its meanings " Moreover, if meaning is a practical consequence, it would differ from individual to individual because practical consequences depend upon an individual's activity. And, hence, meaning would loose its objective character which is essential for any communication.

Def. 10. Some have introduced the word 'means' into their writings as a synonym for 'involves' or 'logically implies'. But what is implied may be a part of the meaning expressed by the implier and is not the whole meaning. The implier and the implied are not equivalent propositions.

Def. 11. Emotions aroused by a thing or proposition are psychological and differ from individual to individual. The same proposition may give rise to different emotions in different individuals. Suppose there is a party of four persons travelling through a forest and someone tells them that 'there are lions in the forest.' In some persons this proposition may give rise to the emotion of fear, while in others to the emotion of courage and bravery. Moreover, the proposition has some objective reference also.

Def. 12. According to this definition, an event is a sign of some other event with which it is connected in some way. Thus the meaning of striking of a match is either a flame or smoke, and the meanings of dreams are unconscious wishes. A proposition through some event means some other event which may be either a physical event (fact) or a psychological event. If it is a physical event, it cannot be accepted as the meaning because, as we have already stated, we know the meaning independently of any fact; if it is a psychological event, it would lead to subjectivism.

Def. 13(a). The Mnemic effects of a stimulus may be psychological or factual. Definitions (b) (c) and (d) can also be interpreted psychologically or factually and are to be rejected for reasons already stated above.

Def. 14 suggests that there is something like *fixed* meaning of a symbol which the users of that symbol ought to be referring.

The fixed meaning is what is often said to be the 'good use' of the symbol. The definition points out that meaning is something objective which all the speakers and hearers of a symbol ought to understand. But the definition is too vague and does not explain and clarify as to what this fixed meaning is.

Definitions 15 and 16 are psychological. 15 is in terms of the mental processes of the speaker, while 16 is in terms of those of the hearer. No psychological definition of meaning can be satisfactory because psychological processes, being subjective, lack the objectivity essential to any communication.

We now attempt to give a more satisfactory definition of the meaning of a proposition. Whenever any proposition is stated three things can be clearly distinguished—(1) Symbols or language through which it is expressed, (2) Idea or thought which it expresses, and (3) Facts, corresponding to the idea, which *may* but *not necessarily*, exist.

From the earliest times men have wondered at the power of symbols. Nearly all our thinking is encompassed by verbal coils. But what is the relation between words, thoughts and facts? "Throughout the whole history of the human race," says Dr. Postgate "there have been no questions which have caused more heart-searchings, tumults, and devastation than questions of the correspondence of words to facts. The mere mention of such words as 'religion', 'patriotism', and 'property' is sufficient to demonstrate this truth. Now, it is the investigation of the nature of the correspondence between word and fact, to use these terms in the widest sense, which is the proper and the highest problem of the science of meaning. That every living word is rooted in facts of our mental consciousness and history it would be impossible to gainsay; but it is a very different matter to determine what these facts may be. The primitive conception is undoubtedly that the name is indicative, or descriptive, of the thing. From which it would follow at once that from the presence of the name you would argue to the existence of the thing. This is the simple conception of the savage."

The view that every proposition expresses a fact is due to the confusion between asserting a proposition and stating a proposition. Every proposition is not asserted, but stated. If a proposition is stated, it does not mean that there is a fact corresponding to it, or, in other words, it is true. Thinking does not take place through

asserted propositions only. Two persons may discuss a problem for a long time without using any asserted proposition. Though both may not know what the facts are, they understand the meaning of each other quite well. If facts were the meaning of a proposition, discussion and thinking could not take place without being in touch with them.

If fact is not the meaning of a proposition, what is it ? It is always an 'idea' or '*thought*'. By 'thought' I do not mean a psychological process in the speaker's or hearer's mind because such a view would lead to subjectivism and defeat the very purpose of communication. The meaning of a proposition is an '*objective* thought'. It is objective in the sense that every speaker and listener will understand the same thing by the proposition. Every proposition expresses an objective thought which is its *fixed meaning*, and, hence, in all fair dealings there is no place for hypocricy, deceit or misunderstanding on account of language. Whenever there is any sincere disagreement regarding the meaning of certain symbols, the fixed meaning is to be searched or developed for good use or communication. But the fixed meaning may give rise to different psychological reactions in different persons. These subjective reactions are to be distinguished from the fixed meaning with which they are associated.

Though we are using the phrase 'meaning of a proposition', this should not be interpreted to mean that a proposition is something different from its meaning. The meaning or content itself is the proposition and the linguistic form by which it is expressed is called a sentence. But every sentence does not express a proposition. Only those sentences which express what can be true or false can be said to express propositions.

Now we shall discuss the relation between propositions and facts. We have already stated that a proposition does not express a fact. It only expresses a meaning or objective thought. Facts corresponding to a meaning may exist or not. If the corresponding facts exist, a proposition is said to be true. Thus between a true proposition and the facts there is the relation of *correspondence.* But such is not the case with false propositions as no corresponding facts exist.

The knowledge of the fact corresponding to a proposition is the consequence of our activity which follows the knowledge of the proposition. In many cases our such activity is immediate so that

we start identifying the meaning with the fact. To us this seems to be the reason for the mistake committed by many thinkers.

Reasoning is a chain of several propositions which are inter-connected by logical relations. It goes on independently of facts, but the logical connections between its different units reflect corresponding connections between facts, *provided* they exist. The main purpose of thinking is to arrive at conclusions corresponding to which facts must exist provided the premises are true and the corresponding factual situation occurs. The *logical structure* of thought corresponds to that of facts. In this sense thought is a mirror of factual reality. The logic of thought is also the logic of factual reality.

Implication is the logical relation between the meanings of the implier and the implied. And since this meaning is an objective thought, implication is an objective relation between the thoughts expressed by the implier and the implied. What is meant by objective relation is that it exists independently of any thinker. The objectivity of implication is the constitutive condition of all inference.

Since facts do not necessarily exist corresponding to every proposition, implication is not a relation between facts. But thought being a mirror of factual reality, if the facts corresponding to the implier and the implied exist, some relation corresponding to implication must also exist. We shall use the word '*containment*' for such a relation. The fact expressed by the implier *contains* the fact expressed by the implied.

The question which is sometimes raised by the logicians is whether the two sides of the relation expressed by entailment (or implication) are distinctively factual in character and whether entailment is a factual relation. If entailment were a factual relation, we could not know it without being in direct contact with facts. But we do know that one proposition entails another without knowing what the facts are. Much of our logical thinking goes on independently of facts. Entailment is not a relation between facts, but between the *meanings* of propositions. The two sides of the relation are not factual but *ideal*. If we clearly understand that a proposition is not factual but ideal, we can easily find a solution to the problem of implication.

In the light of our view that the meaning of a proposition is ideal, Wittgenstein's statement that 'whenever *p* entails *q*, the sense of *q* is contained in that of *p*'[1] can be interpreted to mean that 'whenever *p* entails *q*', the thought expressed by *q* is a part of the thought expressed by *p*'. And such a view seems to be correct. The question may be asked what is meant by a thought being a part of another thought. We may answer that the implier expresses *a particular coherent system of thought* and the implied is its part in the sense that it expresses *one of its constituents*. Implication is the logical relation between a whole particular system of thought and one of its constituents.

Another significant statement of Wittgenstein is that 'if *p* follows from *q* and *q* from *p*, then they are one and the same proposition'[2]. In other words, equivalent propositions are one and the same. We shall call two propositions equivalent when they express the same meaning or thought but in two different forms. For example, the propositions 'an elephant is bigger than a cat' and 'a cat is smaller than an elephant' express the same thought or meaning in two different forms. Anybody will understand the same thing from both of them. Since equivalent propositions have the same meaning, we hold, against Wittgenstein, that they do not entail each other. They are identical. The relation of entailment is different from that of identity (if the latter can be called a relation at all). In entailment the meaning of the implier is somewhat (not totally) different from and wider than the implied.

Just as the same thought can be expressed in two forms, so the same fact can be perceived in two ways. The ways of perceiving the same fact will not make two facts. We can speak of the same fact expressed in two forms as equivalent facts. Really speaking equivalent facts are identical.

In his definition of entailment, already referred to, Malcolm presupposes that entailment expresses a factual relation and he tries to explain this relation. But there are cases in which neither the antecedent nor the consequent expresses a fact, or in which the consequent expresses a fact while the antecedent does not,

1. See *Tractatus*,5.122, p. 75
2. *Ibid.*, 5.141, p. 79.

Consider the entailment 'John owns a mare' entails 'John owns a horse'. Suppose that John does not own a horse, then we have an entailment in which neither the consequent nor the antecedent expresses a fact. Again suppose that John owns a horse but it is not a mare, then we have the entailment in which the consequent expresses a fact while the antecedent does not. Hence, we have to distinguish three cases of the same entailment :

(1) The antecedent and the consequent both express facts. In other words, both are true.
(2) Neither the antecedent nor the consequent expresses a fact, i.e., both are false.
(3) The consequent expresses a fact, while the antecedent does not i.e., the consequent is true, while the antecedent is false.

The fact that entailment holds irrespective of what the factual situations are is sufficient to prove that entailment is not a factual relation. If it were a factual relation, it would have changed in accordence with the factual situations. But it remains the same while the facts change. The truth-values of the component propositions (each of which *by itself* does not assert any entailment-relation and may be symbolized by E^0) are irrelevant to the assertion of the entailment-proposition[1] (which may be symbolized by E^1). Entailment holds and E^1 is asserted in virtue of the meanings of E^0 propositions and not in virtue of their truth-values. In this connexion E. Toms observes, "Perhaps the attempt to explain entailment in terms of fact would not have been made, had logicians followed Russell's advice that they should never concern themselves with the facts, nor with the question whether what they are saying is true. If two statements are statements of the same fact, it certainly follows that the one entails the other but this is only a special case of the entailment of one statement by another. That it is only a special case can at once be seen from the fact that the converse is not true, for if one statement entails another it does not follow that either of them states a fact, so it would not follow that they state the same fact. Entailment may *be a*

1. An entailment-proposition asserts the relations of entailment and is of the form 'if *p*, then *q*'.

consequence of factual connexion but it *is not* factual connexion, and may occur without it."[1] Modifying Russell's views we would like to say that logicians should not concern themselves with facts as such but only with their *logical structure* as it corresponds to thought. The function of logic is to reveal the logical structure of facts. Entailment is not a consequence of factual connexion, nor vice-versa. The relation between the two realms of reality—ideal and factual—is of correspondence and not of entailment. The question may be asked how this correspondence has arisen. The answer is that it is in the *very nature of reality* to have these two corresponding aspects.

In all his examples Malcolm[2] presupposes that entailment-propositions are propositions of facts. Thus he says that the proposition 'This table is brown' entails, and is entailed by 'If anyone were to judge this table to be brown, he would be judging truly'. According to him they are equivalent propositions and express one fact only. But it is obvious that these propositions may be equivalent only if the former is true. There is a difference between stating that 'This table is brown' and that 'This table is brown is true'. The first which just states a proposition may be called a first-order statement, while the second which states the truth-value of the first-order statement may be called a second-order statement. The first-order statement neither entails nor is entailed by its corresponding second-order statement. The former is concerned with the ideal aspect of reality, while the latter with the factual aspect.

Let us consider another example given by Malcolm. He believes that the propositions, 'The average man is 5 ft., 8 inches tall' and 'If the heights of all men were measured, and the figures for the individual heights added together, and if the sum obtained were divided by the number of men measured, the result would be the figure 5 ft. 8 inches', express only one fact. We have already stated that a proposition is independent of any fact, and, therefore, these propositions cannot be said to express the same fact. But they do express the same meaning and must be considered equivalent. Anybody who reads or listens to these propositions wiil understand the same thing, neither more nor less. One

1. 'Fact and Entailment', *Mind*, 1940, pp. 453-54.
2. 'The Nature of Entailment', *Mind*, 1940

may point out that one proposition is short, while the other is lengthy. A lengthy proposition may contain words the meaning of which some may not understand. When we say that two propositions are equivalent what is meant is that anyone with normal intelligence and education will understand the same from both of them. If there is a word which somebody does not follow, that is a personal limitation and does not at all affect the meaning of the proposition. Shortness and length also do not affect the objective meaning of a proposition. The same thought may be compressed or expanded, and, hence, the difference in size arises. In the example under consideration, the thought of 'average height' has been expanded in the lengthy proposition.

Seeing the difficulty of applying Malcolm's definition of entailment to cases in which neither the antecedent nor the consequent expresses a fact, Prior modifies it. The modified definition is "When one proposition entails another, then whatever fact is expressed by the consequent is either identical with or contained in a fact expressed by the antecedent, and whatever fact is denied by the antecedent is either identical with or contained in a fact denied by the consequent."[1] But like a proposition expressing a fact, a proposition denying a fact is a second-order proposition. In the entailment, "'John owns a mare' entails 'John owns a horse'," the constituent propositions neither express nor deny a fact. The proposition 'John owns a mare' is different from the proposition 'John owns a mare is false'. Prior's modified definition cannot be applied to cases of entailment containing first-order propositions.

Let us apply it to second-order propositions. Prior believes that it is applicable in the following cases:

(1) 'John owns a mare is true'entails'John owns a horse is true'.

(2) 'John owns a mare is false' entails 'John owns a horse is false'.

(3) 'John owns a mare is false' entails 'John owns a horse is true'.

The first part of the definition that the fact expressed by the consequent is contained in the fact expressed by the antecedent, is applicable to (1). Prior applies the second part of the definition,

1. 'Facts, Propositions and Entailment', *Mind*, 1948, p. 64.

that the fact denied by the antecedent is contained in the fact denied by the consequent, to (2). It is to be observed here that the order of the relation of containing is reversed here. Instead of the antecedent containing the consequent, the consequent contains the antecedent. If the consequent contains the antecedent, we must say 'John owns a horse is false' entails 'John owns a mare is false'. We cannot deduce or infer 'John owns a horse is false' from 'John owns a mare is false'.

To (3) the first part of the definition is applied. Analysing it Prior says : "If we examine the antecedent more carefully, we shall find that although it is false as a whole, it contains a part that is true, and it is from this part that the consequent follows. 'John owns a mare' is a conjunction of the propositions 'John owns a horse' and 'John's horse is female', and the first of these expresses a fact which enables us to infer a proposition expressing the same fact."[1] Obviously Prior commits a fallacy here. No doubt John owns a mare' is a conjunction of the propositions, 'John owns a horse' and 'John's horse is female', but the proposition 'John owns a mare is false' is not a conjunction of the propositions 'John owns a horse is true' and 'John's horse is female is false'. The proposition 'John owns a horse is true' is not a part of the proposition 'John owns a mare is false'. Both are independent of each other and express different factual situations. Hence we are not justified in inferring one from the other.

(1) is the factual situation corresponding to the relation of entailment. If p entails q, then p is true entails q is true, that is, *the fact P* contains *the fact Q*. But whether p expresses a fact or not cannot be decided on the basis of 'p entails q'. The relation of entailment only expresses the logical structure of thought to which the *logical structure* of facts, *if they exist*, must correspond.

1. 'Facts, Propositions, Entailment', *Mind*, 1948, p. 64.

9

Implication and Necessity

S. Korner[1] rejects Moore's view that 'the proposition that *p* entails *q* is not itself a necessary proposition, but if it is true, then the proposition $p \supset q$ is a necessary proposition'. Strawson,[2] however, seems to support Moore's view, but on grounds different from those of Moore. To examine Moore's view let us have before us the conclusion at which he arrives : "What the dogma of internal relations asserts is that in every case in which a given thing actually has a given relational property, the fact that it has that property is a necessary truth; whereas what I am asserting is that, if the property in question is an 'internal' property, then the fact in question will be a necessary truth, whereas if the property in question is 'external,' then the fact in question will be a mere 'matter of fact.' "[3] Moore holds that not all relational properties are internal, at least some of them are external. If a relational property *Q* is internal to a term *P*, we can say that the proposition '*A* has *P*' entails the proposition '*A* has *Q*', or *p* entails *q*[4] (or *p*

1. See his article 'On Entailment', *Proceedings of the Aristotelian Society*, 1946-47.
2. See his article 'Necessary Propositions and Entailment-Statements', *Mind*, 1948.
3. *Philosophical Studies* (1958), pp. 202-03.
4. On page 284 of *Philosophical Studies*, Moore says that one thing which is always implied by the dogma that 'all relations are internal', is that, in the case of every relational property, it can always be truly asserted of any term A which has that property, that any term which had not had it would necessarily have been different from A. The phrase 'would necessarily have been' is explained by Moore as follows : "To say of a pair of properties P and Q, that any term which had P would neeessarily have had Q, is equivalent to saying that, in every case, from the proposition with regard to any

(Foot-Note Contd.)

entails *q*' is true)[1]; if a relational property *R* is external to the same term, we can say *p* does not entail *r* (or '*p* entails *r*' is false). On the basis of this the conclusion is derived that entailment-statements[2] are not necessary but contingent (true-or-false). But such a conclusion is fallacious. The entailment-statement '*p* entails *q*' is totally different from the statement '*p* entails *r*'. It is not the same statement which is true-or-false but *two different* statements which are true[2] and false respectively. In the former the entailment relation is validly asserted, in the latter it is invalidly asserted. To decide whether an entailment-statement is necessary or not, we have to consider only the first statement. The relation of entailment may be asserted validly or invalidly but we are to consider whether in those cases in which it is validly asserted, it is also asserted *necessarily*. In other words, what we are to consider is whether entailment-statements of the form '*p* entails *q*' are necessary. It should be clear here that an entailment-statement (on entailment-proposition) is that in which the relation of entailment is validly asserted, and expressed by the word 'entails'.

What Moore means by saying that "If '*p* entails *q*' is true, then '$p \supset q$' is a necessary propositlon," seems to be that "If *Q* is an internal relational property of *P*, then any case of *P must* have *Q*." Let us first consider the question whether the statement '*Q* is an internal relational property of *P*' or '*p* entails *q*' is a necessary statement. We often find confusion in the views of those who try to answer this question because they have no clear definition of a necessary statement or proposition before them. A necessary

given term that it has P, it *follows* that that term has Q : '*follows*' being understood in the sense in which from the proposition with regard to any term, that it is a right angle, it *follows* that it is an angle and in which from the proposition with regard to any term that it is red it *follows* that it is coloured." (loc. cit., pp. 284-5)

From the quotation, it is clear that Moore is using the phrase 'would necessarily have' for 'entails'. The proposition 'a relational property P is internal to A' is to mean that 'any term which had not had it would necessarily have been different from A' which is equivalent to the proposition "if A had not had it, A would have been different from what it is' or 'A would necessarily have P' or 'A entails P' which is of the form '*p* entails *q*'.

1. The word 'true' is loosely used with regard to entailment-statements which are really speaking *self-consistent* or *necessary*.
2. For us 'entailment-statement' and 'entailment-proposition' are synonymous phrases,

statement may be defined as that which asserts a necessary relation between its terms or constituents; to deny this relation would be self-contradictory. In the light of this definition, is the statement '*Q* is an internal relational property of *P*' necessary? In accordance with Moore's definition of internal relation, if *Q* is internal to *P*, any term which had not *Q* would necessarily be different from *P*. In other words, an internal property is grounded in the very nature of the term which possesses it. Hence a proposition which asserts the relation between a term and its internal property must be considered to be asserting a necessary relation, and such a proposition is necessary. The assertion of a necessary relation is not only true but absolutely true; it can never be false, to deny it would be self-contradictory. '*p* entails *q*' can never be false. Strawson[1] gives "'X is red' entails 'X is round'" as an example of false entailment-statement. But it is not an entailment-statement, and, therefore, the question of an entailment-statement being false does not arise. We can only say that in the given example the relation of entailment is wrongly asserted and not that it is a false entailment-statement.

Let us now consider another part of Moore's view, viz., " '$p \supset q$' is a necessary proposition" follows from '*p* entails *q*'. Russell has used '$p \supset q$' to mean 'it is not the case that *p* is true and *q* is false'. Moore believes that this proposition is either identical with or logically equivalent to the proposition 'if any thing has *P*, then that thing has *Q*'.[2] But really the two propositions are not logically equivalent but in one sense. '$p \supset q$' has three senses: (1) *p* is true and *q* is true, (2) *p* is false and *q* is true, and (3) *p* is false and *q* is false. The two propositions are logically equivalent only if we interpret $p \supset q$ in the first sense which would be only a *limited* sense and not the whole sense of it. If (1) holds, $p \supset q$ holds; if $p \supset q$ holds, (1) alone does not hold. What Moore means by saying that '$p \supset q$ is necessary', seems to be that there is a necessary relation between the truth-values of *p* and *q* in the first sense, viz. if *p* is true, *q* *must* be true, and this proposition follows from '*p* entails *q*'. I do not see how a factually necessary proposition

1. *loc. cit.*, p. 186.
2. See '*Philosophical Studies*', p. 297. It should be noted that here 'if...then...' does not express the ordinary relation of implication but simply the *occurrence* of *P* and Q *together* without any necessary relation between them.

(which expresses a necessary relation between facts) of the form 'If p is true, q must be true' follows from 'p entails q' unless the latter itself is necessary. A necessary relation between factual propositions[1] is possible only if there is a necessary relation between ideal contents. 'p entails q' expresses a necessary relation between the thoughts expressed by p and q. Corresponding to it there exists a necessary relation between facts, because, as we have already mentioned in the previous chapter, the logical structure of facts corresponds to that of thought. It is in the sense of *correspondence* that we may say that the proposition 'if p is true, q must be true' 'follows' from 'p entails q'.

Korner criticizes Moore's view that the self-same proposition $p \supset q$ is either necessary (if p entails q) or contingent.[2] It seems that both fail to see that the symbolic expression $p \supset q$ does not express only one proposition. The relation of '$\supset$' between p and q holds, as already stated, in three senses or factual situations. Hence $p \supset q$ expresses three factual propositions. Only the first factual proposition, namely, 'p is true and q is true,' is *converted into* a necessary proposition, i. e., 'when p is true, q *must* be true,' *provided* p entails q. '$p \supset q$' is not a necessary proposition, but '$p \supset q$ is necessary' (or 'when p is true, q must be true') expresses a factually necessary proposition. Korner thinks that '$p \supset q$ is necessary' should be regarded as identical with or at least equivalent to 'p entails q.' But these two propositions express the logical structure of different realms of reality—factual and ideal—and, therefore, cannot be regarded as identical. They may be said to be logically equivalent only in the specific sense that the *logical structure* of facts expressed by one *corresponds* to the logical structure of thought expressed by the other.

The necessity involved in any entailment-statement is the necessity of a *particular coherent system of thought*. The logical structure of thought is such that a particular coherent system expressed by the implier must be related to its constituent (the implied) by the relation of implication. Implication is the logical relation between a whole particular system of thought and its constituents and this relation is necessary. If a particular system of thought

1. A factual proposition expresses a fact, i.e., truth or falsity of an empirical proposition.
2. See 'On Entailment', *Proceedings of Aristotelian Society*, 1946-47.

exists, its constituents must also exist. A whole particular system of thought stands by asserting the relation of implication between itself and its constituents. If we deny this relation, the whole given system will fall down.

There are not only systems of thought but also of facts. Corresponding to the ideal necessity, there is the factual necessity which is expressed in statements of the form 'If *p* is true, *q* must be true'. It is this factual necessity which enables us to infer one fact from the other. A factual entailment-statement (i.e., *p* is true entails *q* is true') expresses the necessary relation (for which we have used the term 'containment') between the truth-values of its components. If the relation of entailment were not necessary, we could not infer one fact from the other. Inference is useful because what is inferred necessarily follows from the premises. If what is inferred only follows and does not *necessarily* follow from the premises, then it may also not follow. And if the conclusion may or may not follow from the premises, the inference would not be a valid source of knowledge. If 'three angles of this triangle are equal to two right angles' does not necessarily follow from 'this figure is a triangle', knowledge arrived at by deductive inference would not be valid. As deductive inference does give us valid knowledge, the conclusion must be considered to follow *necessarily* from the premises. In other words, there is a necessary relation between the premises and the conclusion. An entailment-proposition which asserts this necessary relation must be said to be necessary.

Strawson,[1] like Moore, holds that entailment-statements (i.e., statements in which the main verb is 'entails') are not necessary but contingent. They are also intensional statements. He believes that in contingent intensional statements we *use* the main intensional words (such words as 'entails' and when used in this way, 'impossible'. 'consistent', 'necessary', etc., are referred to as 'intensional words' by him) and *mention* the other expressions they contain. These statements are of a higher order than statements in which the expressions mentioned in the contingent intensional statements are used, but not mentioned. Strawson holds that these statements are about the use of expressions, and any statement about the use of expressions is 'contingent', i.e.,'not-

1. *loc. cit.*, pp. 184-85.

(either necessary or self-contradictory). To quote him, "To say that statements of the form " '*p*' entails '*q*' " are contingent makes good sense if statements of this form can be regarded as statements about the expressions occurring within the inner quotation-marks. For no one would deny that in any ordinary sense of the word 'contingent', a statement about the use of an expression is contingent."[1] But here Strawson fails to distinguish between the *specific meaning* or usage any expression (including such phrases as 'strict implication', 'material implication') has in a specific or closed logical system such as that of B. Russell or of C. I. Lewis and the meaning it has in ordinary language. In closed logical systems expressions are given specific meanings or usages which *may change*. So entailment-statements in these systems may be said to tell us something about the usage of the expressions. But in ordinary language expressions related by the word 'entails' have fixed *meanings* and refer to the *very nature of thought* and to the *necessary relation* between its two elements. Their meanings or usage are, in any way, not determined by the entailment-statement. They are pre-determined, but the entailment-statement *asserts* the necessary relation between them. An ordinary entailment-statement, e.g., " 'this chair is red' entails 'this chair is coloured' " is not about the use of the expressions within the inner quotation-marks, but these expressions are used in this statement. It does not tell us anything new about the usages or meanings of the expressions 'this chair is red' and 'this chair is coloured'. Those who state it already know the usages or meanings of these expressions. It does not introduce us to expressions whose meanings we do not previously know. But it gives us knowledge regarding the necessary relation between the meanings or usage of the expressions.

We have criticized the view that entailment-statements are about the use of words or expressions. There are at least two other views put forward by thinkers. One is by Malcolm and Prior who believe that entailment-statements are about facts. But such a view is already rejected by us in the preceding chapter. The other view is expressed by Korner (in his article 'On Entailment', *Proceedings of the Aristotelian Society*, 1946-47) who seems to believe that entailment-propositions[2] are about concepts. But his

1. *loc. cit.*, p. 188.
2. For us 'entailment-statement' and 'entailment-proposition' are synonymous phrases, A 'statement' is a 'proposition' stated or expressed in words written or spoken.

repeated assertion that entailment-propositions determine the logical meaning of propositions or concepts which precede the word 'entails', has inclined Strawson to interpret propositions or concepts as expressions or sentences. Commenting on Korner's views, Strawson says, "But what sense are we to give to the word 'proposition' in the context of this phrase, 'determining the logical meaning of a proposition ?' There is a sense of 'proposition' in which it is correct to say that a proposition may be expressed by a sentence, but in this sense it would be more usual to say that a proposition *is* a meaning than that it has a meaning, and I think it is plain, therefore, that Korner, who speaks of the 'meaning of a proposition', is not using the word 'proposition' in this sense. I think, then, that Korner must be using the phrase 'the meaning of a proposition' as equivalent to something like 'the meaning of all sentences having the same meaning as a given sentence."[1] From this quotation it is clear that Strawson is interpreting the word 'meaning' in the ordinary sense in which it is used when we speak of words, sentences or expressions expressing meanings. It is to be noted that Korner has not used the phrase 'meaning of a proposition', but '*logical* meaning of a proposition'. Strawson seems to have ignored the significant word 'logical' in order to support his own interpretation. If proposition itself is a meaning, logical meaning of a proposition would mean 'logical form of a meaning'. From the views expressed by Korner in the article already mentioned, it is clear that he is not using the phrase 'the *logical* meaning of a proposition' in the sense in which Strawson interprets. What Korner means by the phrase is 'the *logical-form* of a proposition'. This is clear from the following quotation : "The logical meaning of the proposition that this chair is green all over is determined by the entailment- proposition that the proposition that this chair is green all over entails the proposition that it is not red all over. The validity of the entailment-proposition is independent of which object is asserted as having the properties of being green all over and not red all over and of whether it in fact has these properties."[2] Here Korner is trying to distinguish between the logical meaning (or logical form) which is unchangeable and matter or constituents which are changeable and may be replaced

1. Strawson, *loc. cit.*, p. 190.
2. Korner, *loc. cit.*, p. 144

by variables. An entailment-proposition determines the logical form of the proposition which precedes the word 'entails'. Korner's use of the word 'determine' also calls for comment. Strawson gives two meanings of this word : (1) to fix precisely what shall be the case, and (2) to ascertain, to record, exactly what is, or has been, the case. In the light of these meanings, he holds that entailment-statements are 'determinant of usage of expressions'. But such an interpretation (which is Strawson's own view) of Korner's views is wrong as 'determining the logical meaning' does not mean 'determining the usage' Korner's view seems to be that entailment-proposition determines the logical form of a proposition (which precedes the word 'entails') in the sense that entailment holds in virtue of *the logical* form of a proposition. It ascertains or records the logical form and is not concerned with the content of a proposition. It makes no difference whether we say that the logical form of a proposition determines the relation of entailment or entailment determines the logical form of a proposition. A change in either of these will result in the corresponding change in the other. This is what is meant when Korner speaks (*loc. cit.*, p. 150) of the adoption of a new entailment-proposition resulting in 'a change in the *logical* meaning of concepts'. Here the phrase 'a change in the logical-meaning of concepts' does not refer to 'a change in the meaning of an expression', but to 'a change in the logical form of a concept', i.e., to 'a change in the *essential nature* of a concept'. When an entailment-proposition changes, logical meaning and the expression also change. In ordinary language it is not the same expression that can have two different meanings in two different entailment-propositions. So entailment-propositions do not 'determine' (in the sense Strawson interprets this word) the meaning or usage of any expression and are not contingent.

Korner seems to be correct in holding that entailment-propositions are about the logical form of concepts or propositions. They do not describe or endorse any usage. They do not teach us anything about the usage of words or sentences, for those who use them know sufficiently well about it. Nor are they practical exercises (the view that they are practical exercises is suggested by Strawson on p. 189 in his article already mentioned when he says, '*knowing* how to use an expression is not the same as being able to say *how it is used*' , for both those who speak or hear them

have sufficient training in the technique of usage. The stage of thinking or expressing in entailment-statements is a later stage than that of learning. The purpose of using entailment-statements is not to describe or endorse any usage but to express something about the logical form of propositions or concepts.

But the phrase 'to express something about the logical form of propositions' needs some clarification. Korner holds that an entailment-proposition determines the logical form of the proposition preceding the word 'entails'. If it determines the logical form of the proposition preceding the word 'entails', does it also not determine the logical form of the proposition which follows the word 'entails?' We hold that it does so. We cannot put each and every proposition in place of the consequent in an entailment-proposition. Only a particular proposition or propositions 'follow' from any other proposition. There is a definite relation between the logical form of the consequent and that of the antecedent. The purpose of entailment-statements (or entailment-propositions) is to express this relation. But, as we have discussed in chapter VII, there does not exist anything like pure logical form of a proposition devoid of its matter or determined constituents. The meaning of a proposition consists of both—its form and matter. Hence it would be better and sound enough if we say that an entailment-statement *expresses the logical relation*, between the antecedent and the consequent, which holds in virtue of their *meanings*.

As entailment-statements are not descriptive statements about the usage of expressions, we reject Strawson's contention that they are contingent. They express the logical (necessary) relation between the implier and the implied, and, therefore, are necessary. They are necessary in the sense that their truth does not depend upon any fact or thing or usage of expression but upon the logical relation(between concepts) which they express. A logical relation is a necessary relation because it holds in virtue of the essential structure of thought. It holds absolutely between two elements related by it. A proposition which expresses a necessary relation is itself necessary. What it expresses holds absolutely.

Strawson thinks that the assertion that entailment-statements are contingent together with the assertion that necessary statements (or their contradictories) neither entail nor are entailed by any statement, has certain advantages. One advantage is that it avoids the paradoxes of implication. He amends the definition of strict

implication and puts it as follows: " '*p* entails *q*' is to be used to mean...'*p*.not-*q* is impossible and neither *p* nor *q*, nor either of their contradictories, is necessary'." This amended definition may avoid the paradoxes of strict implication. But how are we ever justified in supposing that when *p* entails *q*, neither *p* nor *q*, nor either of their contradictories, is necessary? The cure is worse than the disease. Are not necessary propositions related to each other by the relation of entailment? The denial that a necessary proposition can entail or be entailed by any proposition (and, denial that necessary propositions can be related to each other by the entailment-relation) is too high a price to pay for the solution of paradoxes which can be avoided by other sound definition of entailment. None can deny that entailment-statements are related to each other by the relation of entailment, and they are, as we have already stated, necessary. Any deductive system precisely consists of such statements related to each other by the relation of entailment.

Even if we accept that in specific or closed deductive systems, entailment-statements are about the use of expressions, the usages or meanings of these expressions in a *particular* closed deductive system are *fixed* and cannot change. An entailment-statement within a particular system asserts a necessary relation between these fixed meanings, and, therefore, it must be considered to be necessary with regard to that system. Expressions used in the entailment-statement may change their meanings *outside* that system and the entailment-relation may not hold. So the entailment-statement may be said to be contingent only *with reference to something outside* that system. But it would be better not to use the word 'contingent' for it because it is not true-or-false, in ordinary sense. But Strawson expresses an opposite view. He says: "Deductive systems do not consist of necessary propositions related by the relation of entailment. They consist of contingent intensional statements *to the effect* that such-and-such expressions express necessary propositions; and it is these contingent intensional statements that are related to one another by the relation of entailment. One expression expressing a necessary proposition does not necessitate another such expression; but one expression's *membership* of a system of expressions expressing necessary propositions may necessitate another expression's membership of that system." (*loc.cit.*, pp. 187-88.) Strawson is making a distinction between

the statement '*p*' (which is necessary) and the statement that '*p* is necessary' in a deductive system. While the former is a necessary statement, the latter is an intensional contingent statement. I do not see how the statement that '*p* is necessary' is not itself necessary when '*p*' is necessary. In what sense is '*p*' necessary in a system ? It is necessary in the sense that it can never be denied in that system. If it can never be denied, the statement that '*p* is necessary' can also never be denied in that system. And further the statement that " '*p* is necessary' entails '*q* is necessary' " can never be denied. Strawson seems to accept that any deductive system consists of such propositions. He says : where '*p*' and '*q*' are necessary, " '*p*' entails '*q*' " has no sense but " '*p* is necessary' entails '*q* is necessary' " has a perfectly good sense and may be true. (loc. cit. p. 188).

Strawson wrongly thinks that the statement '*p* is necessary' is informative about the use of the expression '*p*'. The same view is expressed by him on p. 192 (loc. cit.) when he says that the statement " 'If your parents have a second child' , 'you will be a sibling', is logically necessary", is about the usage of the expressions occurring in it. As the proposition 'if your parents have a second child, you will be a sibling', is not about the usage of expressions, but about the thought expressed, I do not see how another proposition which asserts it to be necessary can be about the usage. When p^2 asserts that an entailment-proposition p^1 is necessary, what is meant is that the thought expressed by p^1 is necessary or there is a necessary connexion between the two components of p^1. p^2 does not assert anything about the use of the expression occurring in p^1.

If '*p*' occurs in a system as a necessary statement, we can as well say that the statement '*p* is necessary' occurs in that system as a necessary statement, for we can easily substitute the latter for the former. If '*p*' is a necessary statement, then the statement that '*p* is necessary' is equivalent to the statement '*p*'. The latter only makes explicit what is implicit in the former. Another example will make this point clear. If we say 'if *p*, then *q*', it is a necessary statement, and everybody would understand it as such. But we can as well say " 'if *p*, then *q*' is necessary" which is epuivalent to the former.

Again on p. 193 (loc. cit.) Strawson says, "Necessary statements generally, though the declaration that they are necessary is

informative (or sometimes, misinformative) do not themselves inform : they are formulae to be repeated as an aid to working, verbal exercises for getting in practice for inference, or occasionally moves in the game called 'constructing a deductive system' ". From this quotation it appears as if Strawson is inclined to accept that there are expressions occurring within a deductive system to which the word 'necessary' can be applied. I do not know what these necessary expressions can be apart from the E^0 or E^1, E^2...... propositions[1] which may be expressed in the form '*p* is necessary'.

There seems to be some confusion in Strawson's views regarding the nature of the statement that '*p* is necessary'. The question is whether it is about the proposition (meaning or thought) '*p*' or *expression* '*p*'. If it is about the expression *p*, then to say that the use of an expression is necessary in a deductive system is meaningless, for no expression is necessary in any deductive system (unless it occurs in a formalised meta-language which is about the usage of language). Words, written or spoken, apart from what they express (meaning or proposition) are mere abstract entities. and I do not see how the use of these abstract entities can be necessary in any deductive system. If an expression *p* occurs, it occurs as expressing the proposition or meaning '*p*'. An expression *p* *expressing* the proposition *p* may be said to be the statement *p*. It is only the *statement* '*p*' that cen be necessary in a deductive system. Hence the statement that '*p* is necessary' is not about the expression '*p*', but about the statement '*p*'. And it makes good sense if we say that a statement is necessary in a particular deductive system.

While criticising Strawson, Hampshire[2] suggests that these necessary statements are not statements, but rules or formulae, which are not true-or-false, but observed or not observed. The reason for not calling them statements but formulae seems to be that they are not true-or-false like the meta-statements about the expressions or like the true-or-false ordinary statements about the

1. A proposition which occurs as the antecedent or the consequent, without asserting any entailment-relation by itself, in an entailment-proposition, may be said to be E^0 proposition. An entailment proposition which consists of E^0 proposition is E^1 proposition;an entailment-proposition which consists of E^1 propositions is E^2 proposition, and so on.
2. See his article 'Mr. Strawson on Necessary Propositions and Entailment-Statements', *Mind*, 1948.

the non-linguistic world. No doubt they are not true-or-false like other statements but even so they are statements or propositions, for they express what is *necessarily* true (or what *necessarily holds*) in a particular system. They must be observed and there is no reason why they should not be observed.

Since entailment-statements are not true-or-false like ordinary statements about the empirical world, thinkers are misled to believe that either they are no statements or are statements about the usage or expressions. If we say that an expression (as an abstract entity devoid of meaning) follows from another, it conveys no sense. But it conveys sufficiently good sense if we say that a proposition or statement follows from the other. The invariable relation between an expression and what it expresses (meaning or proposition) is responsible for the failure on the part of thinkers to distinguish between the two and has given rise to the wrong view that entailment-statements are about expressions. E^1 proposition asserts the logical relation between the throughts or concept expressed by E^0 propositions. It is *necessarily* true in the sense that the logical relation of entailment does hold absolutely between E^0 propositions or concepts independently of any fact.

When Lewis defines '$p \prec q$' as 'it is impossible that p is true and q is false', it is clear that he is referring to the propositions p and q and not the expressions p and q, for expressions are neither true nor false. On p. 161 of Symbolic Logic, he distinguishes between relative and absolute meanings of 'possible', 'impossible' and 'necessary'. Relative meaning 'has reference to the relation which the proposition or thing considered has to some state of affairs', while the absolute meaning 'concerns only the relation which the fact or proposition has to itself or to its negative—*what can be analyzed* out of the proposition by sheer logic'. From this also it is clear that Lewis is referring to the *propositions* p and q and not expressions p and q. When he calls his system 'a canon and critique of deductive inference', what he seems to mean is that it can be applied to cases of ordinary deductive inference (though, as we have stated in chapter VI, he could not fully succeed in his attempt). If entailment-statements are about expression, what is the utility of inference? We rarely make inference to know the use of any expression.

It is because entailment-statements assert the *necessary relation*

between the *meanings* of two propositions[1] that they are necessary. Those systems in which entailment-statements occur as such are intensional, while those systems in which the relation of implication (which is not really implication) holds in virtue of the truth-values of the propositions, are extensional. Nelson and Lewis[2] tried to expound intensional systems, whereas Russell gave an extensional system of material implication.

In constructing a deductive system, Lewis's aim was not only to give a 'closed intensional set' which is descriptive of the language-system within which expressions like 'entails', 'necessary', 'consistent', impossible', etc., are used, but also to construct a system which can be the 'canon and critique of deductive inference'. His aim was not only to give a set of rules regarding the use of these expressions, but also to give rules in accordance with which deductive inference from one proposition to another goes on. Rules of deductive inference are not the rules of the specific use of these expressions. If the rules are consistent, that is, form a system, that does not mean that the system is regarding the use of the expressions which occur in it. Even if a *pure* system of meta-language (i.e., a system in which totally new definitions or interpretations are given of all its fundamental or premitive expressions) is possible, Lewis's system is not as such, for he tries to use consistently the ordinary notion of implication. All his theorems refer not only to the expressions of meta-language, but also to the relation of implication between propositons. How far he has succeeded in his attempt to explain implication is a different question. Even if he has failed, that would not justify to say that he constructcd a *pure* system of meta-language, for he tries to use the ordinary notion of implication. Many logicians fail to see that even in a formalized meta-language a definition is given so as to make it conform to prior notions about the meaning of the term as it is used in ordinary language. A formalized meta-language does not only define new symbols, but also explicates some of the already familiar concepts and expressions. Uncertainties or differences regarding the meaning of these expressions make possible different systems of

1. As logicians have often used the phrase 'meaning of proposition' I am also using it but, as I have made it clear *above*, meaning itself is the proposition.
2. We have already stated in chapter VI that Lewis could not fully succeed in this attempt.

meta-language in which the same expression occurs with different meanings. Such systems of meta-language are, really speaking, not totally about the use of any expression or symbols, but also about the exact meaning of different expressions and concepts of ordinary language. They often try to describe the rules in accordance with which reasoning in ordinary language would go on or to define what relationship holds between propositions of ordinary language in terms of concepts and expressions as defined in them.

Are entailment propositions analytic?

If entailment propositions are necessary, the question may be asked whether they are analytic, for it is generally held that necessary propositions are analytic. But the word 'analytic' is over-burdened with many senses, and we have to inquire in what sense, if any, entailment propositions are analytic. One sense of saying that entailment propositions are necessary may be that they assert that a *conjunction* necessarily entails one of its members, that is, they are of the form " 'P_1, P_2 . . . and Pn' entails P_1". In this sense the implier is like a *mathematical sum* of many propositions one of which is the implied. This may be stated in another form '*pq* entails *p*', i.e., a conjunctive proposition entails one of its members. But we have already rejected it as an assertion of entailment in chapter VII. *pq* does not entail *p*, but the latter is identical with one of the members of the former. The relation of entailment is different from that of identity (if the latter can be called a relation at all) between a conjunctive component of the implier and the implied.

That entailment-propositions are not analytic in the above sense can be seen by considering the cases of entailment which depend upon the transitivity of a relation. Let R designate a transitive relation, and 'a' 'b' 'c' three different terms of propositions. Then we can say that 'the propositions aRb and bRc jointly entail the proposition aRc' . . . (1). Let us compare it with '*pq* entails *p*' . . . (2). While the validity of (1) depends upon the transitivity of R, the validity of (2) does not depend upon the transitivity of any relation which *p* and *q* may assert. Hence it is clear that (1) and (2) are of different forms. It would not be proper to call (2) as a case of entailment.

If entailment propositions were analytic in the conjunctive sense we are discussing, then the 'principle of syllogism' would be

redundant since the first member of its antecedent alone would entail its consequent. Take, for example, if *p* entails *q* and *q* entails *r*, then *p* entails *r*. Now *q* would be equivalent to *r* plus some proposition, say, *X*, and *p* would be equivalent to *q* plus X_1. Hence *p* would be equivalent to *r* plus *X* plus X_1. Thus *p* alone would entail *r*.

From the foregoing discussion it is clear that the antecedent of an entailment-proposition is *not a mathematical sum* of propositions one of which is the consequent, and that entailment propositions are not analytic in this sense.

But there is another sense in which they can rightly be called analytic. This sense is that the antecedent of an entailment proposition expresses a system whose constituent is the consequent. It is analytic because it analyses a system into its constituents. The difference between a member of a mathematical sum and that of a system is that the remainder of a mathematical sum can stand significantly without any of its members, while a system as a whole falls down in the absence of any of its members. The relation between the members of the former is only extrinsic, while between those of the latter is intrinsic.

Does a necessary proposition entail its own necessity ?

This is an important question both about necessity and about entailment. If '*p*' is necessary, does p entail that " '*p*' is necessary"? C. Lewy, in his article 'Entailment and Necessary Propositions'[1], has given a negative answer to it. Let us begin with a slightly modified version of the question he posed: Does the proposition "All brothers are male" (B) entail the proposition that "It is necessary that all brothers are male" (N)? It is clear that he is not using 'entail' to mean 'strictly imply'. Lewis tried to identify entailment and strict implication and held 'that the relation of strict implication expresses precisely that relation which holds when valid deduction is possible' and that the system of strict implication explicated those properties of the relation which hold between '*p*' and '*q*' when '*q*' is deducible from '*p*'. There are serious criticisms against this which we have already dealt with in chapter VI and we need not repeat them here. Suffice it to say that if the question whether (B) entails (N) were the question whether (B) strictly implies (N), then the former must strictly imply the latter because

1. This article is published in *Philosophical Analysis* (1963), edited by Max Black.

it is a well-known paradox of strict implication that a necessary proposition is strictly implied by any proposition and Lewy holds that the proposition "It is necessary that all brothers are male" is itself necessary, its denial being self-inconsistent. But he thinks that two further conditions must be satisfied for (B)'s entailing (N), and that (B) is not related to (N), as to satisfy these conditions. For any proposition '*p*' to entail any proposition '*q*' it is necessary "(1) that the propositional function 'R counts in favour of '*q*' should strictly imply the propositional function 'R counts in favour of *p*' and (2) that the propositional function 'R counts against *q*' should strictly imply the propositional function 'R counts against *p*' ". These conditions are laid down to eliminate the possibility of the paradoxes of strict implication. If for *p* to entail *q*, what counts in favour of *p* must count in favour of *q*, then an entailment cannot hold between propositions which are materially, in respect of content, irrelevant to each other, but which are such that if *q* is necessary, it is strictly implied by any proposition *p*. These conditions, though necessary for *p* to entail *q*, are not sufficient, but do eliminate the possibility of the paradoxes of strict implication. For example, if it is necessary that "It is necessary that all brothers are male", then it will be strictly implied by (1) "Mrs. Indira Gandhi is the Prime Minister of India", (2) "Some brothers are not male", and (3) "Fifty-five per cent students of Lucknow University have passed this year". But any evidence which would count in favour of any of these could certainly not be said to count in favour of "It is necessary that all brothers are male", for such an evidence is entirely irrelevant to it. Hence, in accordance with Lewy's conditions, the proposition "It is necessary that all brothers are male" could not be entailed by any of the three propositions mentioned.

He thinks that the proposition "It is necessary that all brothers are male", though strictly implied by "All brothers are male", is not entailed by it, for an evidence could be supposed in favour of the latter which would not count in favour of the former. The evidence is as follows: that all siblings are male, i.e., a woman whose first child is female can have no other children and if her first child is male all subsequent children are male—so that all siblings are male. This hypothetical matter of fact, Lewy thinks, would count in favour of "All brothers are male" but not in favour of "It is necessary that all brothers are male". It should be

noted that "All siblings are male" (S), as established by Lewy, is an hypothetical empirical proposition. It describes a particular state of affairs. There may not be only one but many such facts to support that "brothers are male". But can the necessary proposition "All brothers are male" be said to have been derived from these facts ? It is one thing to say that there are facts *in favour of* or *corresponding* to a necessary proposition and it is another thing to say that a necessary proposition *is derived from* these facts. While the former is true, the latter is not. We may say that there is evidence in favour of a necessary proposition in the sense that there are instances of facts corresponding to it, but there is no evidence in favour of it in the ordinary sense, that is, its truth is not based upon and is not in any way related to these facts. It is not an inductive generalization on the basis of contingent (empirical) propositions but is true a priori or necessary. So it can he said that the hypothetical empirical fact 'all siblings are male' does not count in favour of or is not an evidence for the proposition "all brothers are male' just as this fact is not an evidence for the proposition 'it is necessary that all brothers are male'. Hence, Lewy's criterion when interpreted and applied correctly fails to prove that the former proposition does not entail the latter.

Another important question which deserves consideration is whether a necessary proposition can be entailed by a contingent one. Lewy holds that the contingent proposition "All siblings are male" is an evidence for the necessary proposition "All brothers are male". But this is to make a stretched or unusual use of the word 'evidence'. This is not 'evidence' in the ordinary sense according to which 'Rāma, Mohan and Krishna... who are members of parliament are honest' is the evidence for 'Many members of the parliament are honest". Any empirical reason is irrelevant to the truth of a necessary proposition. Therefore, an empirical proposition does not 'entail' a necessary proposition in the sense of 'being an evidence for' it. Moreover, the relation of 'being an evidence for' is often different from that of 'entailing'. The premises of an inductive inference are evidences for the conclusion but cannot be said to entail it. In many cases the relation of 'being an evidence for' is probable, while the relation of entailment

is always necessary. Where the evidence is complete,[1] we may say that the premises entail the conclusion. But in such case both the premises and the conclusion are empirical propositions. In other words, a contingent proposition entails another contingent proposition.

The truth of a necessary proposition is not at all determined by an empirical proposition. What is often meant by saying that a proposition is necessary is that matter of fact is no ground for its truth. As Wittgenstein said of tautologies, "They have no truth-conditions, for they are unconditionally true". For example, the function "*p* v ∼*p*" is true under all conditions, that is, its so-called truth-conditions are not conditions at all. The truth or falsity of *p* is no evidence for "*p* v ∼*p*". We do not argue for the truth of "*p* v ∼*p*" by saying, "*p* is true, and this justfies concluding that '*p* v ∼*p*' is true". In other words, we do not deduce the truth of "*p* v ∼*p*" from a matter of fact.

But some scholars may hesitate to deny that '*p*' entails '*p* v ∼*p*', or that 'all siblings are male' (taken in Lewy's sense) entails 'all brothers are male'. And then this would be an unusual usage of the word 'entails'. As Quine comments : "...there are philosophers who have through use and custom, grown to feel...at home with the notion of entailment which so pervades G.E. Moore's philosophical analysis. There are philosophers of ordinary language who have grown so inured to the philosophical terms 'entails' and 'inconsistent' as to look upon them, perhaps, as ordinary language. But the reader without such benefits of use and custom is apt to feel... (an) insecurity over these notions ...I contend that the term 'entails' is familiar enough to count as part of the educated person's ordinary vocabulary, where it means merely the converse of the familiar 'follows from'. The insecurity about it lies instead in the fact that it can occur in an unusual setting, e.g., in '*p* entails *q*' where '*p*' is contingent and '*q*' is necessary. How in a given case are we to answer the question, 'Does *p* entail *q*' " ? I think the answer is quite simple and clear. So far as the ordinary sense of the word entailment or implication is concerned, a contingent

1. Evidence is complete in *Summary Induction* (Johnson's term) of the form :
   S_1, S_2, S_3......Sn is P.
   Every S is either S_1 or S_2 or S_3...... or Sn.
   Therefore, every S is P.

proposition cannot be said to imply a necessary one, for the former is irrelevant to the truth of the latter which is true a priori.

An argument may be advanced in favour of the view that '*p*' entails '*p* v ~*q*', and 'all siblings are male' entails 'all brothers are male'. If we were to define '*p* entails *q*' to mean that '*p* is logically inconsistent with the contradictory of *q*', then it may be said that between '*p*' and '~(*p* v ~*p*)', and between 'all siblings are male' and 'all or some brothers are not male', there is apparently a logical inconsistency. But it should be noted that the question of consistency arises only between propositions which belong to the same sphere of meaning, i.e., whose meanings affect each other. There is no consistency or inconsistency between propositions which are independent of each other. The propositions '*p*' and '(*p* v ~*p*)', and the propositions 'all siblings are male' and 'all brothers are male' belong to different spheres of meaning. '*p*' and 'All siblings are male' are empirical propositions, while '*p* v ~*p*' and 'All brothers are male' are analytic propositions. The meaning of an analytic proposition is not determined by that of an empirical one, nor vice versa. An analytic proposition analyses one of its terms, while an empirical proposition is about a state of affairs. Therefore, there is no consistency or inconsistency between '*p*' and '(*p* v ~*p*)' and between 'All siblings are male' and 'All brothers are male'. The apparent consistency is because a necessary proposition, though independent of empirical propositions, can have instances of facts corresponding to it. But these facts do not determine it. What is true a priori or necessary has instances of facts corresponding to it.

A contingent proposition always refers to some empirical situation and it is this situation which makes it true or false. But its meaning and truth-value cannot determine the meaning and *a priori* truth of a necessary proposition because the former refers to the factual realm and the latter to the ideal realm of reality, and these two realms, though corresponding, are independent of each other.

Now having denied Lewy's view that 'all siblings are male' is the evidence for 'all brothers are male' and is not the evidence for 'it is necessary that all brothers are male', we also deny that *on the basis of his condition and argument* we can arrive at the conclusion that " 'all brothers are male' does *not entail* 'it is necessary that all brothers are male'."

But at the same time we do maintain that the former (B) does not entail the latter (N), for they are equivalent propositions. Anybody would understand the same from both the propositions. Necessity is, to use Moore's term, an 'internal property', and a proposition such as 'all brothers are male' cannot fail to have this property. If one were to understand the proposition 'all brothers are male', he would also understand that it is necessary. In saying (N), one has not said anything over and above what is said in saying (B). If (B) is necessary, has one not uttered a redundancy in saying that it is so? The phrase 'is necessary' is redundant in the same way as 'unmarried' in 'unmarried bachelor'.

But it may be argued that understanding a proposition is different from knowing its truth-value. Examples may be cited from empirical propositions. We understand an empirical proposition without knowing whether it is true or false. Can it not be said with regard to analytic propositions that we understand them without knowing their truth or necessity. If we understand the truth[1] of an analytic proposition as soon as we understand the proposition, what is the use of demonstration in mathematics? But it should be noted that the way in which the truth of an empirical proposition is determined is different from the sense in which an analytic proposition is *demonstrated.* The truth of an empirical proposition can be known by sense-experience, while the necessity of an analytic proposition can be demonstrated by analysing it. Every-step or proposition in the process of demonstration is known immediately to be necessary *by insight*. The proof in mathematics only *clarifies* but does not add anything to the necessary nature of a proposition. Necessity is an *intrinsic* property of analytic propositions, while truth-values are *extrinsic* properties of empirical propositions.

1. It should be noted that as the writers have been loosely using the word 'true' with regard to analytic propositions, I have also sometimes used it as such. But, as explained in chapter VII, the appropriate word is 'self-consistent' (or 'necessary').

10

Conclusion: Systematic Intensional Implication

The theory which I have been trying to develop in the present work is the theory of Systematic Intensional Implication which is what is *ordinarily* meant by implication when we say that in any deductive inference the premises *imply* the conclusion. This theory is different from the linear implication of the traditional logicians, the absolute implication of the idealistic logicians and the extensional implication of Russell and Lewis.

The traditional logicians were mainly concerned to keep the independence of terms in any syllogistic reasoning. For them, there is no intrinsic connexion between terms, and in the absence of any such connexion they could not successfully explain how we necessarily pass from the connexions between terms in the premises to the connexion between terms in the conclusion. Nor could they explain the acquisition of the major premise on which the whole syllogistic reasoning is based. Moreover, there are forms of deductive reasoning, e.g., *a fortiori* arguments[1], which cannot be properly reduced to the syllogistic form and to which the *Dictum de omni et nullo* does not apply. How are we to explain the validity of such type of reasoning?

Seeing the difficulties in the traditional theory, the idealistic logicians developed the theory of *absolute*[2] systematic implication.

1. Relational arguments in which the relation is transitive and asymmetrical are sometimes known as *a fortiori* arguments. The following is such an example :

   A is greater than B,
   B is greater than C,
   therefore, A is greater than C.

2. Bosanquet has used the phrase 'systematic implication', but I prefer to use the pharse '*absolute* systematic implication' to distinguish it from my theory of 'systematic intensionali mplication'.

According to it the terms are not indepeneent units but are constituents of the absolute system. Being parts of the same system they are internally related and modify each other. Given certain premises we can tell what the conclusion *must* be, for they are related to each other internally. Mind does not copy or apprehend any ready-made reality but constructs it in accordance with the law of coherence. Those elements which fit into the system are accepted as its parts, while those which do not fit are either modified or rejected. We can never deny any of its parts, for the whole system will fall in the absence of any of them. The conclusion which follows from the premises is a part of the *absolute* system. The whole system will be shattered if we deny it. The *necessity* of the conclusion *following* from the premises (in other words, the necessary character of the relation of implication between the premises and the conclusion) is best expressed in the formula 'this or nothing'. There is the special given system which together with the whole system of reality implies a certain conclusion, and this conclusion must be admitted if we affirm the whole system of reality. The special given system refers to the whole system of reality as it is its part. The particular conclusion is really a conclusion from the whole reality. If we deny the conclusion, we must either modify or deny our whole view of reality.

The idealistic view that a proposition is related to every other proposition of the universe does not appeal to reason and common sense. If this view were correct, one would be able to know everything about the universe by just knowing something about it. In other words, everybody would have become omniscient. But we all know how limited our knowledge is. No man can ever claim to have known the universe fully. It is a mere mystery to us. Even the Indian philosophical systems which speak of absolute knowledge admit that such knowledge is not ordinary. The absolute knowledge (omniscience) of a realised soul is not logical; it may be supra-logical. The logic of the Absolute is unknown to man. How a superman sees one in many and many in one is a mere mystery to ordinary human beings. It is this mystery which is expressed by Bradley when he says that the plurality of relative things *somehow* merges in the Absolute. Śaṅkara expresses it by the word 'Māyā'. The relation between Brahman and the world, between one and many, is Māyā. It is a mystery which baffles all

human intellect and can never be explained. Māyā sets the limit to human knowledge. We are masters of the bundles of relatives but can never go beyond them. We can arrange and re-arrange them and put-forth all our endeavour in doing so but can never grasp the absolute. The logic of the relative is given to us, but the Logic of the Absolute is hidden from us.

If a proposition were related to every other proposition, it would have been possible to construct a logical system showing the logical relations between different propositions of the universe. But none has attempted to do so, nor is it possible, for, really speaking, there are many systems in the universe, not one, which are independent of each other. To try to search for internal relation between the victory of Congress (I) and the good rains this year is futile. Though an astrologer may try to establish some relation between such phenomena, the relation is mere dogma so far as logic is concerned.

The view that there are many systems in the universe is quite consistent with our ordinary experiences and modes of reasoning. We see that the political system is quite different from the weather-system, the family-system is different from the clock-system and so on. The premises and conclusion of one system are irrelevant to and independent of another system. To make this point clear let us take an example of the political system and compare it with the weather-system. The following is an example of the political system :

Russia helps all communist nations,
North Vietnam is a communist nation,
Therefore, Russia helps North Vietnam.

The following is an example of a weather-system :

Monsoon becomes active in the month of July,
This is the month of July,
Therefore, Monsoon has now become active.

I do not see how any internal relation can be established between these two systems or how one can modify the other. We can assert any one of them without having any knowledge of the other. The fact that both are true together is mere co-incidence. The truth of

one does not in any way affect the truth of the other. Even if one were false, the other would remain true by itself.

It may be argued that a proposition is related to every other proposition in the sense that it is different from every other proposition. Since 'all determination is negation', every proposition involves a reference to 'the rest of the universe'. But the relation of 'being different' is not a relation at all and certainly cannot be called an internal relation. 'To be different' expresses the absence of any positive relation and the independence of the things concerned.

For deduction there are inummerable systems, not one. Whenever we deduce, our reasoning moves in a particular system. It is the very nature of thought to move logically, and it is this logical movement that makes a system. Every case of deduction may be said to express a *deductive thought-system.* The different parts of this system are not just put together haphazard but are connected with each other by logical relations.

The chief characteristic of a deductive thought system is that it is *coherent.* Its different elements are related to each other by relations that are not merely conjunctive but internal. Any of its elements determines every other element in it. If any element were taken out, we would have to modify the whole system. The whole particular system would be shattered and would not remain what it is, if we deny any of its elements. Every element, therefore, is *necessary* to it.

The following are the conditions to which every deductive thought-system must conform :

Given a deductive thought-system S, then

(1) If A and B are both constituents in S, then A determines B and conversely.

(2) If A and B are both constituents in S, then A and B are compatible.

(3) If A is a constituent and determines C, then C is a constituent of S.

(4) If A does not determine D nor conversely, then D is not a constituent of S.

(5) If A is a constituent of S, then S *cannot exist* without A. But A *can exist* without S.

Or, when A is a constituent of S, if we assert S, we must also assert A; it would be *self-contradictory* to assert S and deny A,

that is, 'S. not-A' would be impossible, but A can be asserted without asserting S.

If an element is not included in a particular system, it does not mean that both are incompatible with each other. They are said to be *independent* of each other. Since no system is all-inclusive, there would always be elements not included in a particular system.

Implication is the logical relation between a particular system of thought and one of its constituents. If we admit the system, its constituents must also be admitted. To deny any one of them would be to deny the whole system. It stands or falls by any of its constituents.

The relation of implication is necessary for it is the *logical link* between a system and its constituents. The constituents are necessary elements in a system without which it would not be what it is. In any deductive reasoning the premises express a particular thought-system and the conclusion expresses one of its constituents. The conclusion *necessarily* follows from the premises.

All implication is in the form of a hypothetical statement : if p, then q. Here the propositions p and q are *not asserted* but only *stated.* We do not know whether they are true or false. The propositions which state truth-values are different from and of a higher order than the propositions which are merely stated. For example, the proposition 'p is true' is different from the proposition 'p'. Propositions of the former type may be called '*factual* propositions', for they either assert or deny a fact. The propositions which are *about* the empirical world are called empirical propositions and are to be distinguished from the factual propositions. The former are about the empirical world (and do not state anything regarding facts), while the latter either assert or deny a fact. 'This cloth is red' is an empirical proposition, while " 'this cloth is red' is true" is a factual proposition.

Besides these two types of propositions—factual and empirical—there is the third type, the analytic. All implication-statements (which assert the relation of implication) belong to this third type. An analytic statement analyses a concept and brings out its consequence. An implication-statement asserts the relation of implication between a concept (or a particular system of thought) and its constituents or consequences. It is analytic in the sense that we get the constituents by *logically* analysing the concept. Logical analysis is different from mathematical analysis. The former consists in

analysing a system into its different constituents which are mutually connected by logical (necessary) relations, while the latter consists in analysing a whole into its different units which are *conjunctively* combined. Any form of deductive reasoning is an example of logical analysis. '4=2+2' is an example of mathematical analysis. The units of a mathematical whole are not internally related and do not modify each other, while the constituents of a coherent system are internally related and modify each other.

How do we apprehend the logical connexion (i.e., implication) between a particular system of thought and its constituents ? Cook Wilson explains it by *immediate apprehension of the factual connexion.* There are necessary connexions between facts and the mind directly apprehends them. Implication is not known by a priori insight into the way in which knowledge must grow, but is apprehension of a factual connexion. Inference is a series of immediate apprehensions of reality; the conclusion represents an apprehension of reality distinct from the apprehensions expressed in the premises.

Against Cook Wilson it may be asked, what is the guarantee of the validity of these apprehensions ? If reality is known only through them, we can never know reality as it is in itself. We directly know only our thoughts and not any external reality. If there is any necessity in thought it must be self-explained. It cannot be explained by any external reality. This is possible only if we accept that it is in the very nature of thought to move logically. Its logical (necessary) character is not determined by external facts but by its own laws.

All *logical* relations are *a priori.* Their knowledge does not depend upon any factual experience. We know them by mere *insight into the nature of thought* itself. Every man with normal intelligence and understanding does possess such insight.

It may be asked that if one can see logical relations by a priori insight, what is the utility of demonstrating a theorem ? Why should a mathematician give an elaborate proof ? A mathematician does not add anything to our inherent capacity to have a priori insight, but merely teaches us to utilise or manifest the potentiality in a particular way. His task is pedagogical.

Since implication is a logical relation we know it by a priori insight; all hypothetical reasoning proceeds by this insight, no knowledge of facts is necessary for it. The premises and the conclusion do not express facts (unless they are factual propositions),

but a system of thought and its constituents respectively. Implication is of the form '*if p, then q*', while inference is of the from '*p* is *true*, therefore, *q* is *true*'. In the former the propositions are merely stated, in the latter they are asserted. Inference is based on implication which is its constitutive condition.

What a proposition states is its meaning. But that does not mean that a proposition is something different from its meaning. Meaning itself is the proposition. Though a sentence can be distinguished from the proposition which it expresses, no distinction can be maintained between the proposition and the meaning which it expresses. 'Proposition' and meaning' are synonymous words.

The meaning of a proposition is always a particular thought. As explained above (Chap. VIII), by 'thought' I do not mean any psychical content in the thinker's mind, for this would lead to subjectivism. The meaning of a proposition is an *objective* thought. It is objective in the sense that everybody would understand the same thing by it. If it were a subjective thought, it would defeat the very purpose of communication. What we express conveys the same meaning to everybody.

If fact were the meaning of a proposition, we could not understand it without knowing the fact. But we do understand a proposition without knowing what the facts are. We understand what the proposition 'The Great Wall of China is 1259 miles in length' means without reference to any fact. If the fact corresponding to a proposition exists we call it true; if such a fact does not exist we call it false. It is only truth-values and not meanings of propositions which depend upon facts.

Implication is a logical relation between meanings of the propositions expressed by the implier and the implied. Since meaning is objective, implication is also an objective relation. It exists between propositions independent of any individual's activity or thinking. As distinct from it inference is a pyschological process which depends upon an individual's activity or thinking. Inference is based on implication.

The view that implication holds in virtue of the connexion between meanings of propositions is *intensional* and is to be distinguished from the *extensional* view of Russell according to whom implication holds in virtue of the truth-values of the propositions. $p \supset q . = . \sim p \vee q$. *p* materially implies *q* is defined to mean 'either *p*

is false or q is true'. According to this definition implication holds in the following cases:

(1) p is true and q is true
(2) p is false and q is true
(3) p is false and q is false.

It does not hold when p is true and q is false. Implication, being defined in terms of truth-values, can hold between any propositions, provided they conform to the above truth-table. It can hold even between propositions which are independent of each other. Such a view leads to the well-known paradoxes:

(1) A false proposition implies every proposition.
(2) A true proposition is implied by every proposition.

These paradoxes are the natural outcome of the extensional definition. Truth-values are extrinsic properties of propositions. Therefore, any relation which is defined in terms of these values cannot be necessary. A necessary relation can hold only in virtue of the content, essence, intension or meaning of propositions.

Implication is such a necessary relation. What follows from the premises follows necessarily. Therefore, it can hold only between propositions which are connected to each other in virtue of their meanings. It cannot hold between propositions which are mutually independent.

The horse-shoe ($\supset$) is consistently used in the *Principia Mathemetica* and there is nothing wrong if we regard it as a mere abbreviation for '$\sim p \vee q$'. Russell, defending his use of 'implies' in a sense in which no one had ever used it, says, "Provided our use of words is consistent it matters little how we define them."[1] But the use of a word already familiar in a particular sense to express something different is likely to cause confusion and is apt to lead to unfortunate consequences. Russell himself has not always been able to keep the two meanings of 'implies'—ordinary implication and material implication —distinct. He seems to have been led by the association of these two meanings of the word 'implies' into what Moore calls the 'enormous howler' of supposing that 'q can be deduced from p'means the same as 'p materially implies q'. We may quote Russell himself in support

1. *Introduction to Mathematical Philosophy*, p. 146.

of it: "The relation in virtue of which it is possible for us validly to infer is what I call material implication."[1] But, as we have discussed in chapter V, material implication cannot be identified with ordinary implication.

Lewis *tries* to give an intensional definition of implication:

$$p \prec q . = . \sim \Diamond (p \sim q)^{2}$$

That is, '*p* strictly implies *q*' means that 'it is impossible for *p* to be true and *q* false'. Since $q \prec q$ is defined in terms of logical impossibility, it is a narrow, or strict, meaning of 'implies'. Clarifying symbols which he uses, Lewis says, 'the meanings which we symbolize are *absolute*; they concern only the relation which the fact or proposition has to itself or to its negative—what can be analysed out of the proposition by sheer logic.'[3] What can be analysed out of the proposition by sheer logic, is a part of the whole meaning of a proposition. *If* the definition of strict implication refers to the *necessary connexion between the truth-values* of *p* and *q* in virtue of the necessary connexion between their meanings, it may be interpreted as a definition of 'containment' which is a factual relation and corresponds to implication. Whenever, *the fact P* contains *the fact Q*, it is impossible that *the fact P* exists, while *the fact Q* does not exist. But Lewis while applying the definition to different theorems uses it in another sense. We may recapitulate the two meanings of the definition which I have mentioned in chapter VI: (1) It is impossible that *p* is true and *q* false *in virtue of the logical connexion between their meanings or essences*, and (2) It is impossible that *p* is true and *q* false *in virtue of their truth-values* which are *independent* of another. Lewis seems to have used the notion of strict implication in the second sense, for he justifies the paradoxes: (1) An impossible proposition implies every proposition, and (2) A necessary proposition is implied by every proposition. These paradoxes correspond to those of material implication, for Lewis interprets intensional notions: 'impossibility', 'necessity', 'consistency' etc., extensionally. A good example of such an interpretation is the theorem 18.42 which states $\sim \Diamond \sim p \prec p$, i.e., what is necessary is true. What a necessary proposition is apart from its being true, Lewis does not

1. *Principles of Mathematics* (1956), p. 33.
2. Lewis & Langford, *Symbolic Logic*, p. 124.
3. *Ibid.*, p. 161.

tell us. He could not see that 'necessity' (which is logical) always refers to the necessary relation between a coherent system of thought and its constituents. Another example of extensional interpretation is on p. 250 (*Symbolic Logic*) where he takes $p{\sim}p$, that is, 'p is true and p is false' as an impossible proposition.

Truth-values, being extrinsic properties, cannot be analysed out of propositions by sheer logic. Therefore, to be consistent with his own view that he is referring to absolute meaning of notions, Lewis cannot justify the paradoxes on the basis of the truth-values of the propositions which are mutually independent. The essence of a proposition consists in its meaning and only a part of the whole meaning of a proposition can be derived out of it by analysis. While trying to give an intensional definition of strict implication, Lewis runs back into extensional logic. We do not deny that he has consistently used the relation symbolized by '$\prec$' in his system. But this relation is different from and cannot be identified with ordinary implication. Lewis's claim that 'the system of strict implication may be said to provide that canon and critique of deductive inference which is the desideratum of logical investigation',[1] cannot be accepted.

Holding that implication is the logical relation between the meanings of the implier (a particular system of thought) and the implied (one of its constituents), we reject the view of Malcolm[2] that when one proposition entails another, the fact expressed by the consequent is identical with that expressed by the antecedent, or at least is 'not a new fact in addition to' the first. The meaning of a proposition is independent of any fact. We understand the meaning without knowing anything about facts. If both were identical, this could not have been possible. The means of knowing a fact is sense-experience, while that of knowing the meaning of a proposition is a priori understanding. The fact corresponding to a proposition may or may not exist. If there is a fact corresponding to a proposition, it is true; if there is no fact corresponding to it, it is false. Facts make a proposition true or false. Facts are neither true nor false; they simply *are*. It is only propositions which are true or false.

Just as we know the meaning of a proposition without any knowledge of facts, so also we have the knowledge of logical

1. *Symbolic Logic*, p. 247.
2. See his article 'The Nature of Entailment', *Mind*, 1940,

relations between the meanings of propositions without any knowledge of facts. And implication being such a relation we know it independently of any fact. A mere *insight* into the nature of propositions is sufficient to know what they imply. I know that 'if any cloth is green, it is coloured' or 'if all men are mortal and Rām is a man, then Rām is mortal' by mere insight without coming in contact with any fact. Sense-experience is irrelevant to the determination of the relation of implication.

It may be said that there are cases in which both the implier and the implied express facts, and so Malcolm's definition and view can be applied to such cases. But there are also cases of implication in which neither the implier nor the implied expresses a fact, or cases in which the implied expresses a fact, while the implier does not. Take, for example, the implication 'John owns a mare' implies 'John owns a horse'. If as a matter of fact John does not own a horse, then we have an implication in which neither the consequent nor the antecedent expresses a fact. Again, if as a matter of fact John owns a horse but it is not a mare, then we have the implication in which the consequent expresses a fact, while the antecedent does not. Seeing the difficulty of applying Malcolm's definition to such cases, A.N. Prior gives a modified definition: 'When one proposition entails another, then whatever fact is expressed by the consequent is either identical with or contained in a fact expressed by the antecedent, and whatever fact is denied by the antecedent is either identical with or contained in a fact denied by the consequent'.[1]

Both Malcolm and Prior presuppose that implication-propositions are factual propositions which either assert or deny a factual situation. They fail to see that implication-propositions are *analytic* propositions in which the consequent expresses a meaning (thought) which is a part of the meaning (the particular system of thought) expressed by the antecedent. Neither the antecedent nor the consequent expresses or denies any fact. Chains of hypothetical reasoning may be constructed without reference to any fact. A proposition expresses a fact only when it is asserted and denies a fact only when it is stated to be false. But in an implication-statement neither the antecedent nor the consequent is either asserted or denied. They are only stated without any reference to their truth-values.

1. 'Facts, Propositions and Entailment', *Mind*, 1948, p. 64.

The reason for believing that implication-propositions are factual seems to be that there does exist a factual relation corresponding to implication which is ideal. For the corresponding factual relation I am using the word '*containment.*' But correspondence is not identity, nor can it be said to be mutual determination. It is in the very nature of reality to have the two corresponding aspects: ideal and factual. The *logical structure* of thought *corresponds* to that of facts. It is in this sense that thought is a *mirror* of factual reality.

When we state an implication-proposition that does not mean that it expresses a fact or that we are in direct contact with any factual situation. But *if* the corresponding factual situation occurs, its logical structure must correspond to that of the ideal. The fact corresponding to the antecedent *must contain* the fact corresponding to the consequent. In other words, 'when *p* implies *q*, if *p* is true, then *q must* be true'. It may be said to be the '*principle of inference*' as it is on its basis that we infer the truth of *q*, when *p* is true.

There are thinkers, e.g., Nelson and E. Toms, who hold that implication holds in virtue of the formal structures of the propositions concerned. The form remains unchanged throughout the series when the constituents are altered. A proposition may be said to consist of two kinds of constituents: *particular* and *universal.* A universal constituent performs the task of combining other constituents and its form primarily determines the form of a proposition. It cannot be replaced by an individual. A particular constituent occurs only as an individual and can be replaced by another without affecting the form. In the proposition 'Krishṇa loves Rādhā', 'love' is a universal constituent, while 'Krishṇa' and 'Rādhā' are particular constituents.

If implication holds in virtue of the form alone, what is the utility and significance of the particular constituents in a proposition? It should be noted that the meaning of a proposition is not determined by the universal constituents alone but also by the particular ones. In the absence of the latter, a proposition would be reduced to a propositional function. Propositional functions are empty sounds and do not convey any real or concrete meaning. They may be devices to exhibit the forms but are useless to express any meaning by themselves. The real meaning of a proposition consists of both—form and matter—the universal and the particular constituents.

If implication is the relation between the meanings of the implier and the implied, it cannot be the relation between their forms only. It is the relation between the *whole* meanings, consisting of both form and matter, expressed by the implier as well as by the implied. The whole meaning of the implier refers to a particular system of thought which cannot be expressed by mere form. The formal structure, e.g., if A is greater than B and B is greater than C, then A is greater than C, is only a linguistic device to express the form of a class of implication-propositions. In Reality there is no system corresponding to such empty formal structures. Reality consists of concrete systems made up of both universal and particular constituents.

An important question concerning implication-propositions is whether they are contingent (true-or-false) or necessary. According to Moore, 'the proposition that *p* entails *q* is not itself a neccssary proposition, but, if it is true, then the proposition $p \supset q$ is a necessary proposition.'[1] The reason for believing that implication-propositions are contingent seems to be that there are propositions in which the relation of implication is falsely asserted. Strawson, supporting Moore, gives " 'x is red' entails 'x is round' " as an example of false implication-proposition. But it should be noted that it is not an implication-proposition at all, and, therefore, the question of an implication-proposition being false does not arise. " 'x is red' implies 'x is coloured' " is an example of an implication-proposition. It is *necessarily* true, for it can never be denied. It asserts a necessary relation between the implier and the implied.

What Moore means by saying that if *p* entails *q*, then $p \supset q$ is a necessary proposition, seems to be that when *p* implies *q*, there is a necessary connexion between the truth-values of *p* and *q*, that is, *if p* is true, *q must* be true. But such a necessary connexion is possible only if we accept, against Moore[2], that *the fact P* contains *the fact Q*. Corresponding to the necessary relation of implication between propositions, there is the necessary relation of *containment* between facts. Just as there are systems of thought so also there

1. *Philosophical Studies*, pp. 302-03.
2. Moore says : 'I do not see how we can possibly do justice to the facts without supposing that there are genuinely different propositions and genuinely different facts which nevertheless mutually entail one another.' ('Facts and Propositions,' *Proceedings of the Aristotelian Society*, Suppl. Vol. VII, p. 198.)

are systems of facts. The logical structure of ideal systems corresponds to that of factual systems. The necessity of 'if *p*, then *q*' and of 'if *p* is true, then *q* is true', is the necessity of ideal and factual systems respectively.

Strawson[1] holds that implication-statements are *about the use* of expressions, and any statement about the use of expressions is contingent. He fails to distinguish between a statement in which an expression *is used* and a statement which is *about the use* of that expression. An implication-statement " 'this cloth is red' entails 'this cloth is coloured' " is not about the use of the expressions within the inner quotation-marks, but these expressions are used in this statement. It does not tell us anything about the usage or meaning of these expressions, and those who use it already know the usage or meaning of these expressions perfectly well. Implication-propositions are not about the usage of expressions but about the *logical connexion* between thoughts or concepts which they express.

In the light of the above discussions, I now propose to give a satisfactory definition of the notion of implication as it is *ordinarily* used.

'*p* implies *q*' means that 'the meaning (a particular coherent system of thought) expressed by *p* has the meaning expressed by *q* as its constituent'. Using the symbol 'I' for implication and symbol '$\longleftrightarrow\!\!\!\!\!\twoheadrightarrow$' for 'has as its constituent', we can express it as follows:

$$p \, \mathrm{I} \, q. = .p \longleftrightarrow\!\!\!\!\!\twoheadrightarrow q$$

It should be noted that here '*p*' does not symbolize '*p* is true', but only 'the *proposition* (meaning or thought) *p*'. Similarly, '*q*' symbolizes 'the proposition *q*'. Implication, being defined in terms of the meaning and not truth-value, is an intrinsic relation between propositions, and cannot hold between propositions that are independent of one another. To know whether a proposition implies another, we need not know their truth-values, but their logical structure. The logical structure of a proposition can be known by mere *insight* without reference to any fact.

When a proposition *q* is a constituent of the proposition *p*, it would be self-contradictory to state *p* and not to state *q*. In other

1. See his article 'Necessary Propositions and Entailment-Statements', *Mind*, 1948.

words, 'p and $\sim q$'[1] would be *intensionally impossible.* Using the symbol '$i \sim \Diamond$' 'for 'intensional impossibility', we can state the definition of implication in another form:

$$p \text{ I } q .=. i \sim \Diamond (p \sim q)$$

that is, "p implies q" means "the proposition '$p \sim q$' is self-contradictory or intensionally impossible".

The notion of impossibility, as used here, is to be distinguished from that of Lewis. Here 'impossibility' refers to the *impossibility of an ideal system's existence without any of its constituents,* whereas Lewis calls a proposition[2] which is both true and false impossible. For him the impossibility of a proposition can be determined by itself on the basis of its contradictory *truth-values.* For p to strictly imply q, it is sufficient that p be an impossible proposition *by itself,* but for p to *ordinarily* imply q, '$p \sim q$' would be an (intensionally) impossible proposition. It is the extensional interpretation of 'impossibility' which is the source of the paradox that an impossible proposition implies every other proposition. We interpret 'impossibility' intensionally, so no such paradox arises.

If 'p and $\sim q$' is intensionally impossible, then q is a *necessary constituent* of p, that is, the proposition q is necessarily *in p.* Here the necessity refers to the necessary relation between an ideal system and one of its constituents. As against this, Lewis interprets 'necessity' also extensionally. Theorem 18.42 (*Symbolic Logic*) states $\sim \Diamond \sim p \prec p$, that is, what is necessary is true. What a necessary proposition is apart from its being true, Lewis does not tell us. So a necessary proposition must be taken as true by itself without any relation with any other proposition. The denial of a necessary proposition being impossible, there follows the paradox that 'a necessary proposition is implied by every proposition'.

Every case of $p \prec q$ is not a case of p I q, but every case of p I q may be said to be a case of $p \prec q$, *provided* the corresponding factual situation occurs and we interpret 'impossibility' in the definition of strict implication as the 'impossibility of a *factual* system's existence without any of its constituents'. As we have stated in chapter VI, '$p \prec q$' may be interpreted to express a *necessary relation between* the *truth-values* of p and q, but unfortunately Lewis does not interpret it as such. Whenever p I q, there is

1. '$\sim q$' is here used for 'the *proposition* which is contradictory of q'.
2. '$p \sim p$', *Symbolic Logic*, p. 250.

a necessary relation between the truth-values of p and q—it is impossible that p is true and q false (provided the corresponding factual situation occurs). Using the symbols 'fP' and 'fQ' for 'the fact P' and 'the fact Q' respectively, and the symbol 'C' for 'contains,' we can express the logical structure of a system of thought and corresponding facts as follows:

Whenever p I q

$$p \longleftrightarrow q \qquad fP \,\mathrm{C}\, fQ \qquad (1)$$

$$\sim q \longleftrightarrow \sim p \qquad \sim fQ \,\mathrm{C} \sim fP \qquad (2)$$

It can be read as "whenever p implies q, (1) the *thought-system* p has as its constituent the thought q, and the factual system P contains the fact Q; and (2) the thought $\sim q$ 'has as its constituent'[1] the thought $\sim p$, and the fact $\sim Q$ contains the fact $\sim P$."[2]

It should be noted that the logical structure expressed in (2) is the reverse of that expressed in (1). It is the peculiar characteristic of a system that it necessarily includes or contains each of its constituents; and if any of them is negated, the whole system is negated. A system cannot exist without its constituents but a constituent (as an independent element) can exist without the system. The difference between the absolute system and a relative system is that a constituent cannot exist apart from the absolute system, while a constituent, as an independent element or as a part of another system, can exist apart from a relative system. If an element belongs to a system, it is not its exclusive possession. It may belong to many systems simultaneously. A man may be a leader in the political system, a scholar in the educational system, and a business-man in the trade-system; and all these systems may be expressed in the form of different logical systems of ordinary reasoning. The fact that a constituent can exist independently of the system gives rise to the theorem that whenever p I q,

$$q \sim\!\longleftrightarrow p \text{ and } fQ \sim\!\mathrm{C}\, fP$$

that is, the thought q does not have as its constituent the thought p, and the fact Q does not contain the fact P.

1. Here 'has as its constituent' means 'includes.' The field of $\sim q$ is wider than that of q and in some sense includes $\sim p$.
2. It should be noted that I am expressing here only the *logical structure* of facts, *provided* they occur.

Material implication, being defined in terms of the truth-values of the propositions, can hold between propositions which are independent of one another, and this gives rise to the well-known paradoxes. If $p \supset q$, it does not mean p I q. When p I q, '$p \supset q$ is necessary' or 'it is impossible that p is true and q false' or 'the fact P contains the fact Q', provided the factual situation occurs.

In accordance with our definition of implication, p I p does not hold.[1] The relation of p to itself (if it can be called a relation at all) is that of identity, and is to be distinguished from implication which is the relation between a particular system of thought and one of its constituents. The factual relation fP C fP, corresponding to p I p, does not also hold.

As p I p does not hold, pq I p does not also hold. p is identical with one of the two propositions conjunctively combined as pq. pq is the mathematical sum of p and q. It is not a system and does not imply p. Similarly its corresponding factual situation $fP.fQ$ does not contain fP.

In the light of this, if pq I r, then $p \neq q \neq r$;[2] and the corresponding factual situation is, if $fP.fQ$ C fR, then $fP \neq fQ \neq fR$.[3]

1. See *above*, Chap. VII.
2. '$p \neq q \neq r$' is to be read as 'the propositions p, q and r are not identical'.
3. '$fP \neq fQ \neq fR$' is to be read as 'the facts P, Q, and R are not identical'.

Appendix

Arthāpatti and Implication

The Mīmāṃsakas and the Advaitins accept Arthāpatti as a separate Pramāṇa or valid source of knowledge and distinguish it from inference. The etymological meaning of the word 'Arthāpatti' is 'the derivation of a meaning' or 'postulation of a fact' (Arthāpatti=arthasya āpattiḥ; artha=meaning or fact, āpattiḥ=postulation). To understand the nature of arthāpatti, let us consider its different instances given by the thinkers.

A man who is known lo fast by day is still growing fat. Here we find an *apparent contradiction* between his increasing fatness and his fasting. There is a given fact which is to be explained (upapādya). Hence we proceed from the knowledge of something to the knowledge of that which explains it (upapādaka). The only way to explain the apparent contradiction between increasing fatness of Devadatta and his fasting is to suppose that he eats at night. This is not only supposition but *necessary knowledge derived* from the given facts. So scholars, e.g., Dr. S. Radhakrishnan[1] and Prof. S.N. Das Gupta,[2] have translated the word 'arthāpatti' as 'implication' which expresses that the conclusion *necessarily follows* from the premises.

Take another example. A man is known to be alive. But we do not find him at home. Hence we derive the conclusion that he must be somewhere outside his home. It is this knowledge which removes the contradiction between his being alive and his absence from home.

Arthāpatti is of two kinds: Dṛṣṭārthāpatti and Śrutārthāpatti. The first means the necessary supposition of a fact in order to

1. *Indian Philosophy*, Vol. II (1931), p. 393.
2. *A History of Indian Philosophy*, Vol. I (1922), p, 391.

explain *perceived facts*. For example, on the negation of the perceptual judgment, 'this is silver', by a subsequent perceptual judgment, 'this is not silver, but a shell', we have to assume the falsity of the silver that previously appeared to sight.

The second, i.e., Śrutārthāpatti, etymologically means the supposition of a fact in order to explain a fact known through *testimony*. It is, again, of two forms : (1) Abhidhānānupapatti and (2) Abhihitānupapatti. The first concerns the supposition of a verbal expression (abhidhāna). If somebody asks 'to close', we supply the *word* 'door' in order to explain its sense. This is an instance of the first type of śrutārthāpatti. An instance of the second type occurs when on being told by the scriptures that by performing the Jyotiṣṭoma sacrifice one can go to heaven, we suppose that the sacrifice must produce some lasting unperceived merit (Apūrva) without which a momentary sacrifice cannot be the cause of a life in heaven.[1]

We are here mainly concerned with dṛṣṭārthāpatti. What is the relation between the given facts and the derived conclusion in it? Can it be said to be inference? So far as śrutārthāpatti is concerned, it can easily be seen that it is not inference. In the case of abhidhānānupapatti, we only supply the *word* in order *to complete the sentence*. This is just filling in the blank. In the case of abhihitānupapatti, we derive a conclusion not on logical grounds but on the basis of our religious beliefs. A man not brought up in the Hindu traditions, will not derive the conclusion that the Jyotiṣṭoma sacrifice generates some lasting unperceived merit (apūrva) from the statement that a man who performs the Jyotiṣṭoma sacrifice goes to the heaven. Nor will he believe such a statement to be true.

Prabhākara and Kumārila differ regarding the nature of the psychological process in arthāpatti. Prabhākara holds that the perception of Devadatta's absence from home throws into *doubt* the previously acquired knowledge that he is living. It is for the removal of this doubt that we presume that he must be living somewhere outside the house. This necessary presumption (arthāpatti) is different from inference. In inference there is no element of doubt, for it is only when the smoke is perceived to

1. See *Vedānta Paribhāṣā* (1942), Chap. V, pp. 124-26.

exist beyond the least element of doubt that the inference of fire is possible. The testimony of our self-reflective consciousness is also against the supposition that arthāpatti is inference. In none of the above cases of arthāpatti do we say 'I infer' (anuminămi), but on the contrary we say 'I suppose' or 'I assume' (kalpayāmi).[1]

Kumārila, however, criticises this explanation of Prabhākara and gives a different analysis of the process of arthāpatti. If the fact that Devadatta is living is made doubtful by the absence of Devadatta at his home, then the doubt may be removed by the supposition that Devdatta is dead. If it was already known that Devadatta was living and his absence from the house creates doubt, how then can the very fact which created the doubt remove the doubt? So the real procedure of arthāpatti is quite different. There is no doubt about the life of Devadatta. But there is an *apparent contradiction* between his life and his absence from the house, and the mind cannot reconcile these two things without presuming that he must be somewhere outside his house.

A question arises : can arthāpatti be not reduced to inference ? The Bhāṭṭa Mīmāṃsakas point out some fundamental differences between the two. In an inference we *first* know the liṅga or the middle term, e.g., smoke, and afterwards, through it, the liṅgi or the major term, e.g., fire. The two are not known together, but one after the other. But this does not hold good if arthāpatti is put in the form of an inference. If arthāpatti were an inference, what would be the middle term ? When we argue that Devadatta exists outside his house because he is living and yet absent from his house, we cannot take mere 'absence from home' as the middle term, since he may be dead and altogether non-existent; nor can we say that the mere fact of his 'being alive' is the mark (middle term) to infer the fact of his being out, for a living man may live inside his house. We must say, therefore, that 'the fact of his living together with the fact of his absence in the house' is the middle term from which we infer his existence outside the house. But we cannot connect these two facts except through the assumption of his existence outside the house. That is to say, the knowledge of the middle term presupposes already the knowledge of the fact to be proved, and there remains nothing to be inferred. Hence the

1. *Vedānta Paribhāṣā*, Chap. V, p. 127.

attempt to reduce arthāpatti to inference is vitiated by a *petitio principii.*

But it may be asked how this criticism of the Mīmāṃsakas applies to the disjunctive-categorical syllogism to which arthāpatti can be reduced. Taking the example given above, Vācaspati reduces it to the following inference :

'Devadatta, who is alive, is either at home or is out,
Devadatta is not at home,
Therefore, he is out.'[1]

The Mīmāṃsakas may ask how at all we could obtain the major premise, 'Devadatta, who is alive, is either at home or is out'. We are supposed to know *only two* facts, namely that Devadatta is alive and that he is not at home. From these two facts we are to infer that 'he is out'. But the major premise in the inference already contains this conclusion as one of its meanings. Hence the charge of *petitio principii* stands.

The Naiyāyikas reduce arthāpatti to the categorical form of *vyatireki-anumāna.* Can the objection of *petitio principii* be applied to this form also ? The Mīmāṃsakas may reply in the affirmative. The following is the categorical form of the syllogism given above :

The absence of a man from all places outside home, while he is not at home, is a case of the absence of his life;
Devadatta, while he is not at home, is alive;
Therefore, Devadatta is not absent from all places outside home, i.e., he is outside home.

Here again the major premise presents the same problem. It is a universal proposition and contains the knowledge of the conclusion. It asserts the relation of vyāpti or Universal Concomitance between the absence of the major term and the absence of the middle term, and the conclusion is also an indirect instance of this relation. Hence here also we have a *petitio principii.*

The Mīmāṃsakas' argument comes to this : the reduction of arthāpatti to the *syllogistic form* involves a *petitio principii,*

1. In Western logic this type of argument is called *Modus tolledo tollens.*

whereas there is no *petitio principii* in arthāpatti. So arthāpatti cannot be reduced to the syllogistic form. But from this it is wrongly concluded that arthāpatti is not inference. That syllogism is the only form of inference is a wrong view held by the ancient Greek as well as Indian thinkers. The Mīmāṃsakas were right in saying that in arthāpatti there is no universal proposition, corresponding to the major premise of a syllogism. When we conclude that Devadatta is outside his house from the fact that he is living and the fact that he is not at home, no universal proposition or generalization is before us. To reduce this reasoning to the syllogistic form would be to stretch its meaning too far which is not warranted by the given facts. We have before us two facts which seem to be contradictory. The apparent contradiction is because they are parts of a system, and this contradiction can be removed only by making the system complete. By presuming that Devadatta is outside his house, we make a coherent system. The presumption is not arbitrary but *necessarily follows* from the given facts. It is not the work of our imagination but a *logical* conclusion from the data. So we can say that although arthāpatti is not syllogistic, it is a form of inference in which the premises imply the conclusion. It is 'implication' that expresses the logical or necessary relation between the premises and the conclusion. In arthāpatti the premises and the conclusion being factual propositions, we can say that the relation between the *facts expressed* by them is that of *containment.* The premises express a factual system which *appears* to be self-contradictory but is really coherent, and it contains the fact expressed by the conclusion as its constituent.

List of Symbols

Symbol	*Is used for*
$\supset$	Materially implies
⥽	Strictly implies
I	Ordinarily implies (Systematic Intensional Implication)
E	Entails
C	Contains
O	is consistent with
$\Diamond$	is possible
$\sim\Diamond$	is impossible
$i\sim\Diamond$	is intensionally impossible
v	or
$\leftarrow\!\!\twoheadrightarrow$	has as its constituent
$\sim$	(1) not
	(2) is false
$\vdash$	is asserted
$=$	is equal to
$\neq$	is different from
$\therefore$	therefore
fP	the (corresponding) fact P
fQ	the (corresponding) fact Q
fR	the (corresponding) fact R

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Corrigenda

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52	26	that is blue,	that 'it is blue',
54	39	the apprehension	the one apprehension
56	21	rcality	reality
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60	17-18	to infer validity	validly to infer
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99	8	*pEp*	*p* E *p*
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134	4	'$p \vee \sim q$'	'$p \vee \sim p$'
139	10	inummerable	innumerable
142	32	pyschological	psychological
144	8	*q* ⥽ *q*	*p* ⥽ *q*
144	27	of another	of one another

